# MOCE I KOMPENSACJA
# W UKŁADACH Z NIESINUSOIDALNYMI PRZEBIEGAMI
# PRĄDU I NAPIĘCIA

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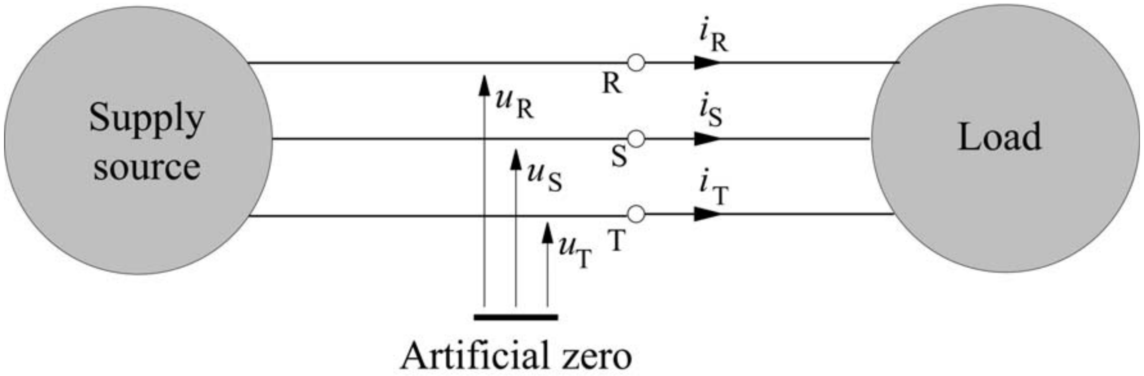
## Moc pozorna w układach trójfazowych, trójprzewodowych




Trzy definicje mocy pozornej:

$$S_A = U_R I_R + U_S I_S + U_T I_T$$

$$S_G = \sqrt{P^2 + Q^2}$$

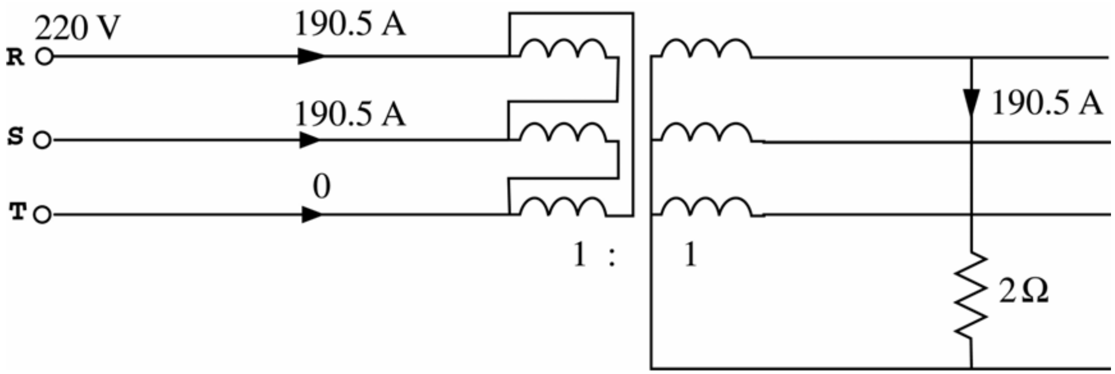
$$S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2}$$

Która jest poprawna?

$P = 72.3$ kW

Ideal

$P = 72.3$ kW




$$S = S_A = U_R I_R + U_S I_S + U_T I_T = 83.8\ \mathrm{kVA}$$

$$S = S_G = \sqrt{P^2 + Q^2} = 72.3\ \mathrm{kVA}$$

$$S = S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\ \sqrt{I_R^2 + I_S^2 + I_T^2} = 220\sqrt{3} \times 190.2\sqrt{2} = 102.7\ \mathrm{kVA}$$

$$\lambda = \frac{P}{S}$$

$$\lambda_A = \frac{P}{S_A} = 0.86 \qquad \lambda_G = \frac{P}{S_G} = 1 \qquad \lambda_B = \frac{P}{S_B} = 0.71$$

**Jaką wartość ma współczynnik mocy?**

Powszechnie stosowane równanie mocy

$$S^2 = P^2 + Q^2$$

z mocą czynną:

$$P \overset{\mathrm{df}}{=} \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T [u_\mathrm{R} i_\mathrm{R} + u_\mathrm{S} i_\mathrm{S} + u_\mathrm{T} i_\mathrm{T}]\,dt = \sum_{f=\mathrm{R,S,T}} U_f I_f \cos\varphi_f$$

i mocą bierną:

$$Q \overset{\mathrm{df}}{=} \sum_{f=\mathrm{R,S,T}} U_f I_f \sin\varphi_f$$

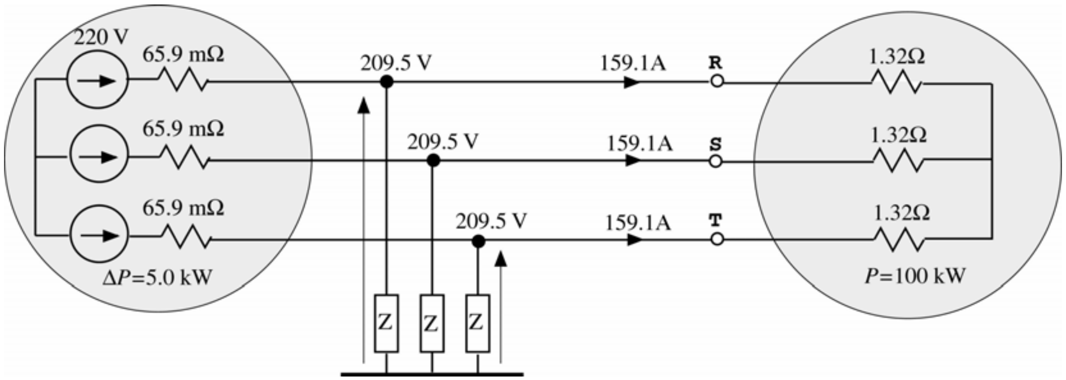
narzuca geometryczną definicję mocy pozornej *S*. Przy innych definicjach równanie mocy nie jest spełnione

Ocena i wybór definicji mocy pozornej $S$
ze względu na straty energii w źródle zasilania i współczynnik mocy:

*L.S. Czarnecki:*

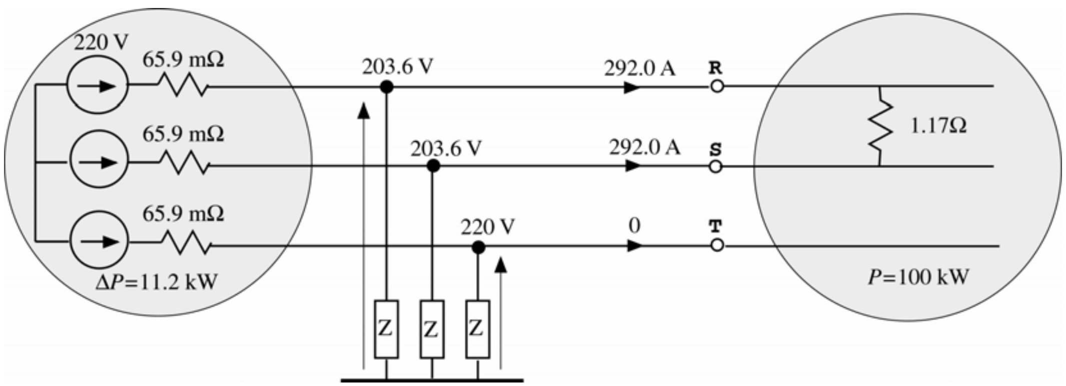
**Energy Flow and Power Phenomena in Electrical Circuits:
Illusions and Reality,**

*Archiv fur Elektrotechnik*, (82), No. 4, pp. 10-15, 1999.




$S_A = S_G = S_B = 100$ kVA

$\lambda_A = \lambda_G = \lambda_B = 1$

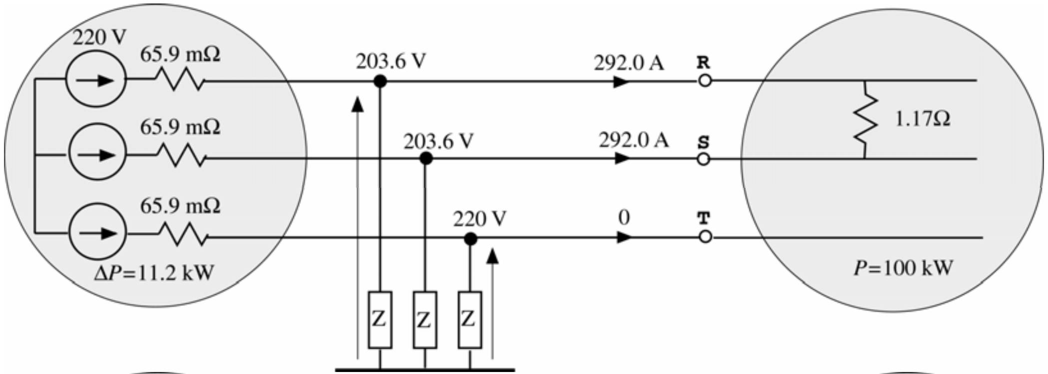



$S_A = 119$ kVA, $\lambda_A = 0.84$

$S_G = 100$ kVA, $\lambda_G = 1$

$S_B = 149$ kVA, $\lambda_B = 0.67$

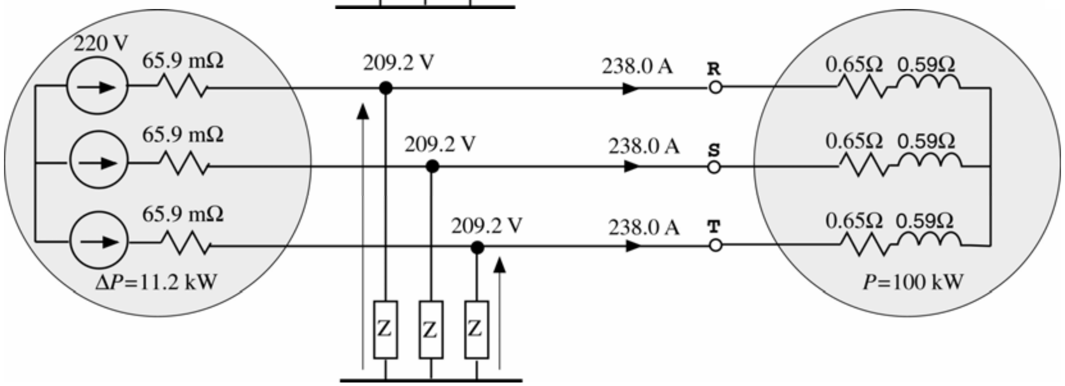
**Definicja geometryczna** $S = \sqrt{P^2 + Q^2}$ **jest błędna**




$S_A = 119$ kVA, $\lambda_A = 0.84$

$S_G = 100$ kVA, $\lambda_G = 1$

$S_B = 149$ kVA, $\lambda_B = 0.67$



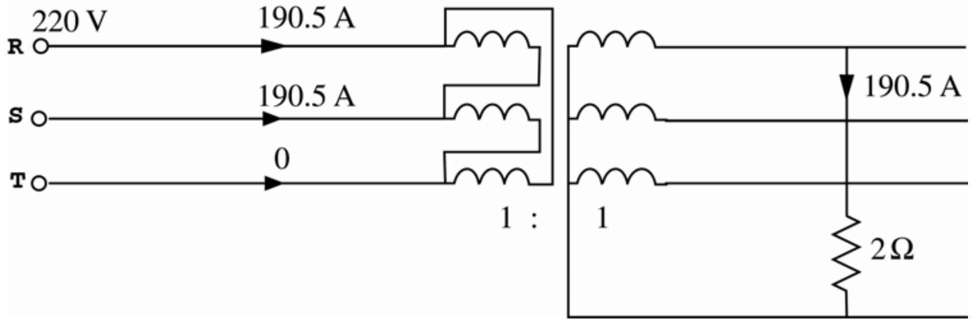

$S_A = S_G = S_B = 149$ kVA

$\lambda_A = \lambda_G = \lambda_B = 0.67$

$$S = \sqrt{U_R^2 + U_S^2 + U_T^2}\,\sqrt{I_R^2 + I_S^2 + I_T^2}$$

**Definicje** $S = U_R I_R + U_S I_S + U_T I_T$, $S = \sqrt{P^2 + Q^2}$ **są błędne**

$P = 72{,}3$ kW,
$Q = 0$




$$S = \sqrt{U_R^2 + U_S^2 + U_T^2}\ \sqrt{I_R^2 + I_S^2 + I_T^2} = 220\sqrt{3} \times 190{,}2\sqrt{2} = 102{,}7\ \text{kVA}$$

## Powszechnie stosowane równanie mocy

$$S^2 = P^2 + Q^2$$

**nie jest spełnione**

**Jeśli równanie**

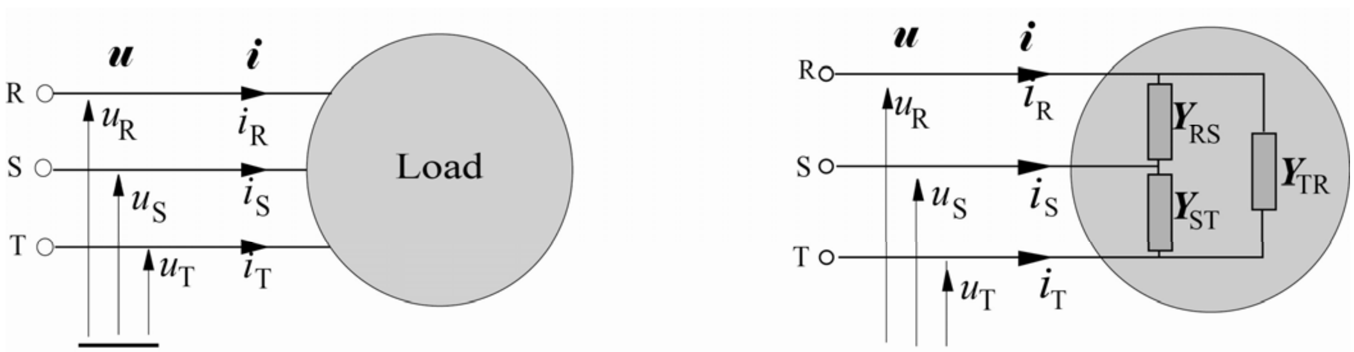
$$S^2 = P^2 + Q^2$$

**jest błędne, trzeba znaleźć nowe równanie mocy**
**!!!**

*L.S. Czarnecki:*

Equivalent Circuits of Unbalanced Loads
Supplied with Symmetrical and Asymmetrical Voltage
and their Identification,
*Archiv fur Elektrotechnik,* 78 pp. 165-168, 1995**.**

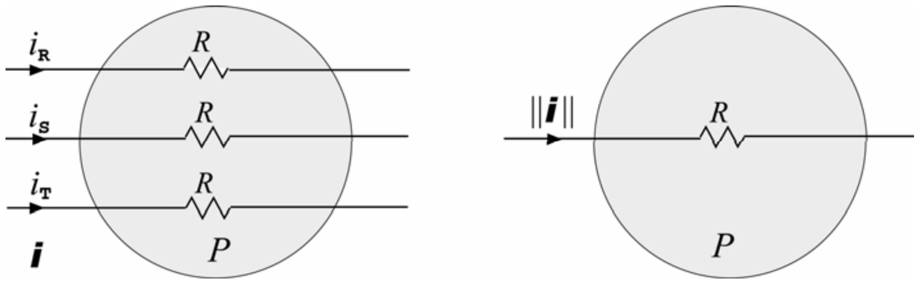
Każdy odbiornik trójfazowy zasilany napięciem sinusoidalnym
ma równoważny odbiornik o konfigracji $\Delta$




Jeśli napięcie jest symetryczne,
każdy odbiornik ma
nieskończenie wiele odbiorników równoważnych o konfigracji $\Delta$

## Wartość skuteczna wektora trójfazowego

$$\boldsymbol{x} \overset{\text{df}}{=} \boldsymbol{x}(t) \overset{\text{df}}{=} \begin{bmatrix} x_{\mathrm{R}}(t) \\ x_{\mathrm{S}}(t) \\ x_{\mathrm{T}}(t) \end{bmatrix} \overset{\text{df}}{=} \begin{bmatrix} x_{\mathrm{R}} \\ x_{\mathrm{S}} \\ x_{\mathrm{T}} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{X}_{\mathrm{R}} \\ \boldsymbol{X}_{\mathrm{S}} \\ \boldsymbol{X}_{\mathrm{T}} \end{bmatrix} e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\,X\,e^{j\omega_1 t}$$




Moc czynna symetrycznego urządzenia trójfazowego

$$P = R\frac{1}{T}\int_0^T (i_{\mathrm{R}}^2 + i_{\mathrm{S}}^2 + i_{\mathrm{T}}^2)\,dt \overset{\text{df}}{=} R\|\boldsymbol{i}\|^2$$

$$\|\boldsymbol{i}\| \overset{\text{df}}{=} \sqrt{\frac{1}{T}\int_0^T (i_{\mathrm{R}}^2 + i_{\mathrm{S}}^2 + i_{\mathrm{T}}^2)\,dt} = \sqrt{\|i_{\mathrm{R}}\|^2 + \|i_{\mathrm{S}}\|^2 + \|i_{\mathrm{T}}\|^2}$$

**jest wartością skuteczna prądu trójfazowego**

*Leszek.S. Czarnecki*:

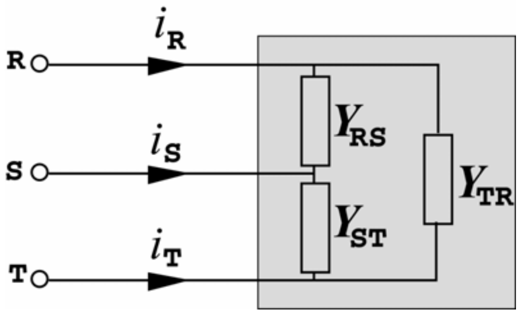
**Orthogonal Decomposition of the Current in a Three-phase Non-linear Asymmetrical Circuit with Nonsinusoidal Voltage**,
*IEEE Transactions on Instrumentation and Measurement,* Vol. IM-37, No. 1, pp. 30-34, 1988.

W obwodach jednofazowych: $S \overset{\mathrm{df}}{=} \|u\|\,\|i\|$

W obwodach trójfazowych: $S \overset{\mathrm{df}}{=} \|\boldsymbol{u}\|\,\|\boldsymbol{i}\|$

Definicja Buchholtz'a dla obwodów z przebiegami sinusoidalnymi

$$S_{\mathrm{B}} = \sqrt{U_{\mathrm{R}}^2 + U_{\mathrm{S}}^2 + U_{\mathrm{T}}^2}\,\sqrt{I_{\mathrm{R}}^2 + I_{\mathrm{S}}^2 + I_{\mathrm{T}}^2}$$




$$\begin{bmatrix} \boldsymbol{U}_\mathrm{R} \\ \boldsymbol{U}_\mathrm{S} \\ \boldsymbol{U}_\mathrm{T} \end{bmatrix} \stackrel{\mathrm{df}}{=} \boldsymbol{U} \qquad \begin{bmatrix} \boldsymbol{U}_\mathrm{R} \\ \boldsymbol{U}_\mathrm{T} \\ \boldsymbol{U}_\mathrm{S} \end{bmatrix} \stackrel{\mathrm{df}}{=} \boldsymbol{U}^{\#}$$

$$\boldsymbol{Y}_\mathrm{RS} + \boldsymbol{Y}_\mathrm{ST} + \boldsymbol{Y}_\mathrm{TR} \stackrel{\mathrm{df}}{=} \boldsymbol{Y}_\mathrm{e} = G_\mathrm{e} + jB_\mathrm{e},$$ Admitancja równoważna

$$-(\boldsymbol{Y}_\mathrm{ST} + \alpha \boldsymbol{Y}_\mathrm{TR} + \alpha^{*} \boldsymbol{Y}_\mathrm{RS}) \stackrel{\mathrm{df}}{=} \boldsymbol{A},$$ Admitancja niezrównoważenia

$$\boldsymbol{i} = \sqrt{2}\,\mathrm{Re}\,\boldsymbol{I}\, e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\{(G_\mathrm{e}\boldsymbol{U} + jB_\mathrm{e}\boldsymbol{U} + \boldsymbol{A}\,\boldsymbol{U}^{\#})e^{j\omega_1 t}\}$$

$$\boldsymbol{i}_\mathrm{a} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{G_\mathrm{e}\boldsymbol{U}\, e^{j\omega_1 t}\}$$ - Prąd czynny

$$\boldsymbol{i}_\mathrm{r} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{ jB_\mathrm{e}\boldsymbol{U}\, e^{j\omega_1 t}\}$$ - Prąd bierny

$$\boldsymbol{i}_\mathrm{u} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}\, e^{j\omega_1 t}\}$$ - Prąd niezrónoważemia (Unbalanced current)

$$\boldsymbol{i} = \boldsymbol{i}_\mathrm{a} + \boldsymbol{i}_\mathrm{r} + \boldsymbol{i}_\mathrm{u}$$

## Ortogonalność wektorów trójfazowych

$$\|\boldsymbol{x}+\boldsymbol{y}\| = \sqrt{\frac{1}{T}\int_0^T [\boldsymbol{x}(t)+\boldsymbol{y}(t)]^2\, dt} = \sqrt{\|\boldsymbol{x}\|^2 + 2(\boldsymbol{x},\boldsymbol{y}) + \|\boldsymbol{y}\|^2}\,.$$

Wektory $\boldsymbol{x}$ i $\boldsymbol{y}$ są wzajemnie ortogonalne jeśli ich iloczyn skalarny jest równy zeru

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{y}(t)\,dt = 0$$

Wówczas:

$$\|\boldsymbol{x}+\boldsymbol{y}\| = \sqrt{\|\boldsymbol{x}\|^2 + \|\boldsymbol{y}\|^2}$$

Iloczyn skalarny $(\boldsymbol{x},\boldsymbol{y})$ można wyznaczyć znając wektory zespolonych wartości skutecznych $X$ i $Y$,

$$(\boldsymbol{x},\boldsymbol{y}) = (x_{\mathrm{R}},y_{\mathrm{R}}) + (x_{\mathrm{S}},y_{\mathrm{S}}) + (x_{\mathrm{T}},y_{\mathrm{T}}) = \mathrm{Re}\{\boldsymbol{X}_{\mathrm{R}}\boldsymbol{Y}_{\mathrm{R}}^{*} + \boldsymbol{X}_{\mathrm{S}}\boldsymbol{Y}_{\mathrm{S}}^{*} + \boldsymbol{X}_{\mathrm{T}}\boldsymbol{Y}_{\mathrm{T}}^{*}\} = \mathrm{Re}\{X^{\mathrm{T}}Y^{*}\}$$

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}$$

$$(\boldsymbol{i}_{\mathrm{a}}, \boldsymbol{i}_{\mathrm{r}}) = \mathrm{Re}\{\boldsymbol{I}_{\mathrm{a}}^{\mathrm{T}}\, \boldsymbol{I}_{\mathrm{r}}^{*}\} = \mathrm{Re}\{G_{\mathrm{e}}\, \boldsymbol{U}^{\mathrm{T}} (jB_{\mathrm{e}}\, \boldsymbol{U})^{*}\} = \mathrm{Re}\{-\, jB_{\mathrm{e}} G_{\mathrm{e}} \|\boldsymbol{u}\|^{2}\} = 0$$

$$(\boldsymbol{i}_{\mathrm{a}}, \boldsymbol{i}_{\mathrm{u}}) = \mathrm{Re}\{\boldsymbol{I}_{\mathrm{a}}^{\mathrm{T}}\, \boldsymbol{I}_{\mathrm{u}}^{*}\} = \mathrm{Re}\{G_{\mathrm{e}}\, \boldsymbol{U}^{\mathrm{T}} (\boldsymbol{A}\, \boldsymbol{U}^{\#})^{*}\} = 0$$

$$(\boldsymbol{i}_{\mathrm{r}}, \boldsymbol{i}_{\mathrm{u}}) = \mathrm{Re}\{\boldsymbol{I}_{\mathrm{r}}^{\mathrm{T}}\, \boldsymbol{I}_{\mathrm{u}}^{*}\} = \mathrm{Re}\{\, jB_{\mathrm{e}}\, \boldsymbol{U}^{\mathrm{T}} (\boldsymbol{A}\, \boldsymbol{U}^{\#})^{*}\} = 0.$$

Wektory prądu czynnego, biernego i prądu niezrównoważenia
są wzajemnie ortogonalne

$$\|\boldsymbol{i}\|^{2} = \|\boldsymbol{i}_{\mathrm{a}}\|^{2} + \|\boldsymbol{i}_{\mathrm{r}}\|^{2} + \|\boldsymbol{i}_{\mathrm{u}}\|^{2}$$

Prądy:

$$\boldsymbol{i}_{\mathrm{a}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\}$$

$$\boldsymbol{i}_{\mathrm{r}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{\,jB_{\mathrm{e}}\,\boldsymbol{U}\,e^{j\omega_1 t}\}$$

$$\boldsymbol{i}_{\mathrm{u}} \stackrel{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\boldsymbol{U}^{\#}\,e^{j\omega_1 t}\}$$

nazywa się

***„składowymi fizycznymi"***

wektora prądu liniowego odbiornika trójfazowego

Teorię mocy opartą na tym rozkładzie, nazwano

**„Teorią mocy składowych fizycznych prądu"**
**Ang.: „Currents' Physical Components (CPC) power theory"**

## Równanie mocy odbiornika trójfazowego zasilanego symetrycznym i sinusoidalnym napięciem

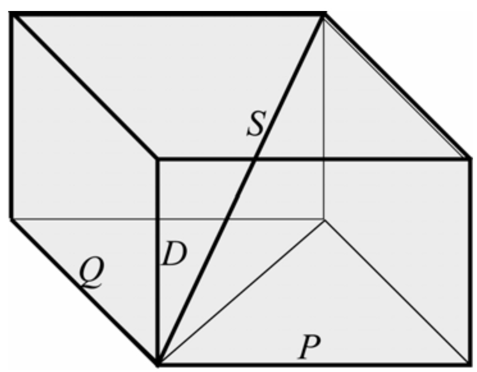
$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}\|^2 \qquad | \times \|\boldsymbol{u}\|^2$$

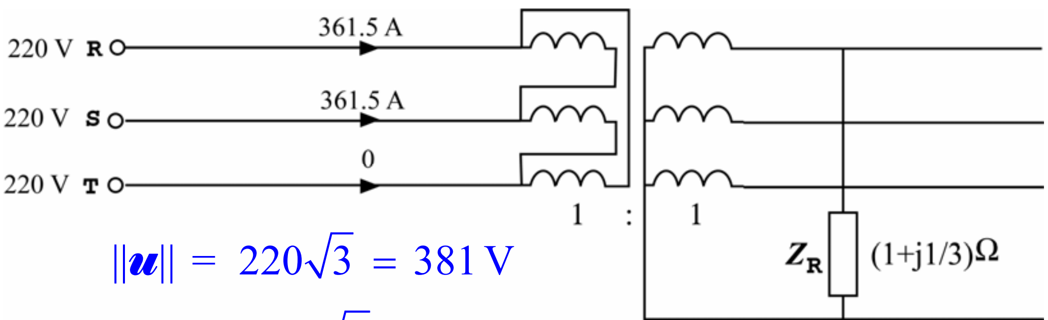
$$S^2 = P^2 + Q^2 + D^2$$

$$P \overset{\mathrm{df}}{=} \|\boldsymbol{i}_\mathrm{a}\|\,\|\boldsymbol{u}\| = G_\mathrm{e}\,\|\boldsymbol{u}\|^2$$ Moc czynna

$$Q \overset{\mathrm{df}}{=} \pm\|\boldsymbol{i}_\mathrm{r}\|\,\|\boldsymbol{u}\| = -B_\mathrm{e}\,\|\boldsymbol{u}\|^2$$ Moc bierna

$$D \overset{\mathrm{df}}{=} \|\boldsymbol{i}_\mathrm{u}\|\,\|\boldsymbol{u}\| = A\,\|\boldsymbol{u}\|^2$$ Moc niezrównoważenia






$$\|\boldsymbol{u}\| = 220\sqrt{3} = 381\text{ V}$$

$$\|\boldsymbol{i}\| = 361{,}5\sqrt{2} = 511\text{ A}$$

$$\boldsymbol{Y}_{\mathrm{RS}} = \frac{1}{\boldsymbol{Z}_{\mathrm{R}}} = 0{,}90 - j\,0{,}30 = 0{,}95\,e^{-j18^0}\ \mathrm{S}$$

$$\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + jB_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} = 0{,}90 - j\,0{,}30\ \mathrm{S}$$

$$\boldsymbol{A} = -\alpha^{*}\boldsymbol{Y}_{\mathrm{RS}} = 0{,}95\,e^{j42^0}\ \mathrm{S}$$

$$\|\boldsymbol{i}_{\mathrm{a}}\| = G_{\mathrm{e}}\|\boldsymbol{u}\| = 0{,}90\times 381 = 343\text{ A}$$

$$\|\boldsymbol{i}_{\mathrm{r}}\| = |B_{\mathrm{e}}|\,\|\boldsymbol{u}\| = 0{,}30\times 381 = 114\text{ A}$$

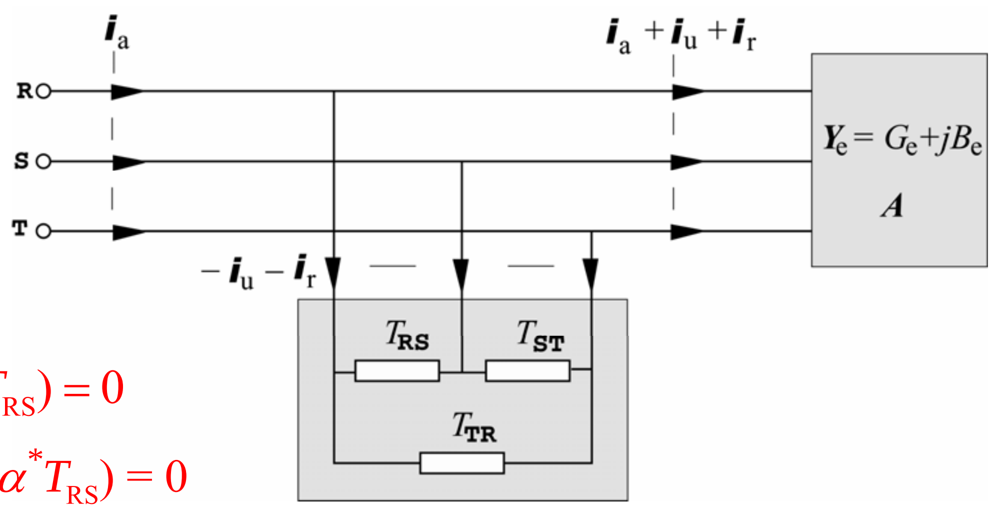
$$\|\boldsymbol{i}_{\mathrm{u}}\| = A\|\boldsymbol{u}\| = 0{,}95\times 381 = 361\text{ A}$$

$$\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}\|^2} = \sqrt{343^2 + 114^2 + 361^2} = 511\text{ A}$$

$S = 195$ kVA, $P = 131$ kW, $Q = 43$ kVAr, $D = 138$ kVA

## Kompensacja prądu biernego i prądu niezrównoważenia

$$\lambda = \frac{P}{S} = \frac{||\boldsymbol{i}_{\mathrm{a}}||}{\sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2}}$$




$$B_{\mathrm{e}} + (T_{\mathrm{ST}} + T_{\mathrm{TR}} + T_{\mathrm{RS}}) = 0$$

$$\boldsymbol{A} - j\,(T_{\mathrm{ST}} + \alpha T_{\mathrm{TR}} + \alpha^{*} T_{\mathrm{RS}}) = 0$$

Rozwiązanie ze względu na susceptancje kompensatora

$$T_{\mathrm{RS}} = (\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_{\mathrm{e}})/3$$

$$T_{\mathrm{ST}} = (2\,\mathrm{Im}\{\boldsymbol{A}\} - B_{\mathrm{e}})/3$$

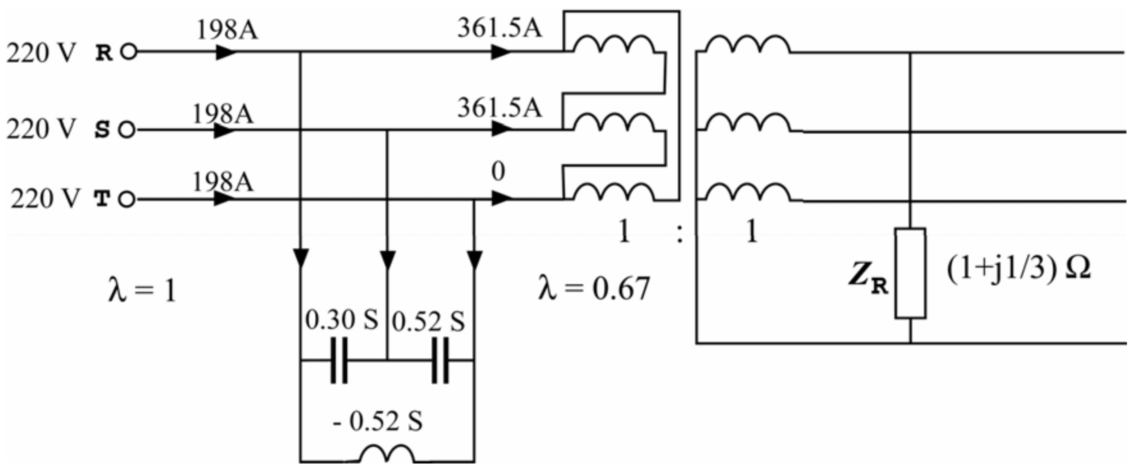
$$T_{\mathrm{TR}} = (-\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_{\mathrm{e}})/3$$

$$T_{RS} = (\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_e)\,/\,3 \;=\; 0.30\ \mathrm{S}$$

$$T_{ST} = (2\,\mathrm{Im}\{\boldsymbol{A}\} - B_e)\,/\,3 \;=\; 0.52\ \mathrm{S}$$

$$T_{TR} = (-\sqrt{3}\,\mathrm{Re}\{\boldsymbol{A}\} - \mathrm{Im}\{\boldsymbol{A}\} - B_e)\,/\,3 \;=\; -0.52\ \mathrm{S}$$

$||\boldsymbol{i}_a|| = 343$ A, $||\boldsymbol{i}_r|| = 114$ A, $||\boldsymbol{i}_u|| = 361$ A




$||\boldsymbol{i}_a|| = 343$ A, $||\boldsymbol{i}_r|| = 0$, $||\boldsymbol{i}_u|| = 0$,

$$S^2 = P^2 + Q^2 + D^2$$

$P \stackrel{\text{df}}{=} ||\boldsymbol{i}_{\mathrm{a}}||\ ||\boldsymbol{u}|| = G_{\mathrm{e}}\ ||\boldsymbol{u}||^2$ Moc czynna

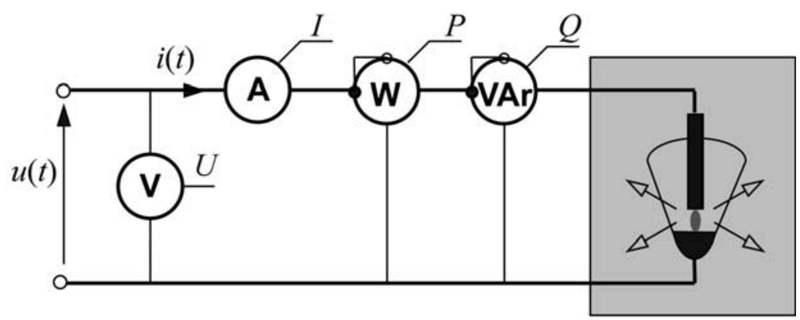
$Q \stackrel{\text{df}}{=} \pm\, ||\boldsymbol{i}_{\mathrm{r}}||\ ||\boldsymbol{u}|| = -\, B_{\mathrm{e}}\ ||\boldsymbol{u}||^2$ Moc bierna

$D \stackrel{\text{df}}{=} ||\boldsymbol{i}_{\mathrm{u}}||\ ||\boldsymbol{u}|| = A\, ||\boldsymbol{u}||^2$ Moc niezrwnoważenia

To równanie mocy jest poprawne tylko
dla odbiornika liniowego, czasowo-niezmienniczego (LTI),
zasilanego napięciem
symetrycznym i sinusoidalnym

Aby je uogólnić na odbiorniki LTI z niesinusoidalnym napięcie zasilania,
niezbędna jest poprawna teoria mocy obwodów jednofazowych
zasilanych niesinusoidalnie.

# Steinmetz experiment: 1892

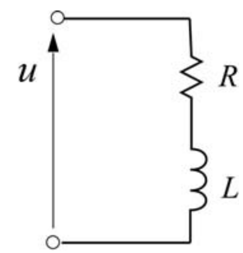



$$P^2 + Q^2 < S^2$$

**?????**

Poprawność równania mocy w warunkach niesinusoidalnych zakwestionował Steinmetz w 1892 roku,

Do lat osiemdziesiątych, po 90 latach rozwoju teorii mocy, jej stan wyglądał jak poniżej




1927: Budeanu: $S^2 = P^2 + Q_B^2 + D^2$ $\quad Q_B \overset{df}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$

1931: Fryze: $S^2 = P^2 + Q_F^2$ $\quad Q_F \overset{df}{=} \|u\|\|i_{rF}\|$

1971: Shepherd: $S^2 = S_R^2 + Q_S^2$ $\quad Q_S \overset{df}{=} ||u||||i_{rS}||$

1975: Kusters: $S^2 = P^2 + Q_K^2 + Q_r^2$ $\quad Q_K \overset{df}{=} ||u||||i_{rC}||$

1979: Depenbrock: $S^2 = P^2 + Q_1^2 + V^2 + N^2$

1984: Czarnecki: $S^2 = P^2 + Q^2 + D_s^2$

**1927**

C.I. Budeanu, Professor of Bucharest University, Romania, introduced definition of the reactive power

$$Q = Q_{\mathrm{B}} \overset{\mathrm{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

$$P^2 + Q_{\mathrm{B}}^2 \leq S^2$$

Budeanu concluded that there is also other power associated with the waveform distortion, and introduced a new power quantity, called

**Distortion Power**

$$D \overset{\mathrm{df}}{=} \sqrt{S^2 - (P^2 + Q_{\mathrm{B}}^2)}$$

Budeanu's power equation has the form:

$$S^2 = P^2 + Q_{\mathrm{B}}^2 + D^2$$

**1987**

**L.S. Czarnecki: What is Wrong with the Budeanu's Concept of Reactive and Distortion Powers and Why it should be Abandoned,**

*IEEE Trans. on Instrumentation and Measurements*

**Question:** Is the Budeanu's Reactive Power a **measure of energy oscillation?**

**For a single harmonic:**

$$u_n(t) = \sqrt{2}\, U_n \cos n\omega_1 t$$

$$i_n(t) = \sqrt{2}\, I_n \cos(n\omega_1 t - \varphi_n)$$

**The instantaneous power:**

$$p_n(t) = \frac{dW_n}{dt} = u_n(t)\, i_n(t) = P_n\,(1+\cos 2n\omega_1 t) + Q_n \sin 2n\omega_1 t$$

$$P_n = U_n I_n \cos\varphi_n \qquad Q_n = U_n I_n \sin\varphi_n$$

**$Q_n$ is an amplitude of the energy oscillation**

**At distorted voltages and currents:**

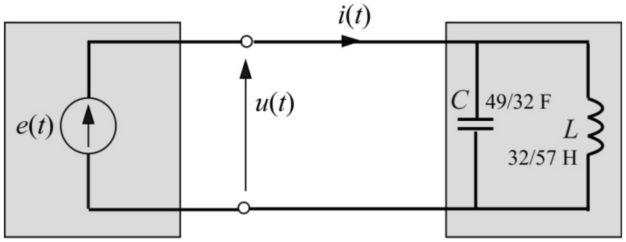
$$u(t) = \sum_{n=0}^{N} u_n(t), \qquad i(t) = \sum_{n=0}^{N} i_n(t) \quad \Rightarrow \quad Q_B = \sum_{n=1}^{N} U_n I_n \sin\varphi_n = \sum_{n=1}^{N} Q_n$$

$$Q_n \gtrless 0, \quad Q_B \text{ could be equal to zero, even if } Q_n \neq 0$$

**Budeanu's Reactive Power is no measure of energy oscillation**

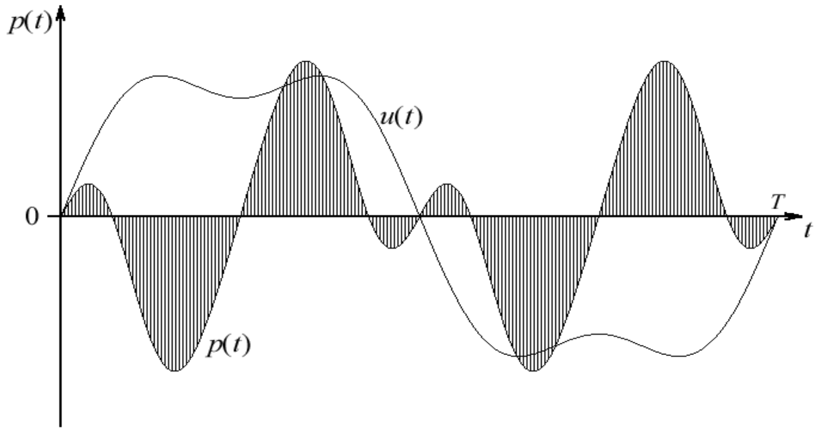
# Why Budeanu definition of reactive power Q is wrong?

$$u(t) = \sqrt{2}(100 \sin \omega_1 t + 25 \sin 3\omega_1 t)\ \text{V}$$




$$i(t) = \sqrt{2}\ [25 \sin(\omega_1 t - 90^0) + 100 \sin(3\omega_1 t + 90^0)]\ \text{A}$$

$$Q = \sum_{n=1}^{\infty} U_n I_n \sin \varphi_n = 100 \times 25 \times 1 + 00 \times 25 \times (\text{-}1) = 0$$




**There are energy oscillations in spite of zero Budeanu’s reactive power *Q***

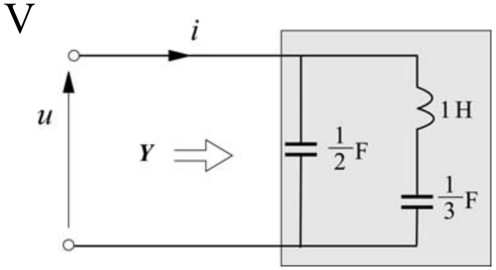
# Why Budeanu's definition of Distortion power D is wrong?

$$D \overset{\text{df}}{=} \sqrt{S^2 - P^2 - Q^2} = \sqrt{\frac{1}{2} \sum_{r \in N} \sum_{s \in N} U_r^2 U_s^2 \, |\mathbf{Y}_r - \mathbf{Y}_s|^2}$$

$D = 0$ if for each r, s:

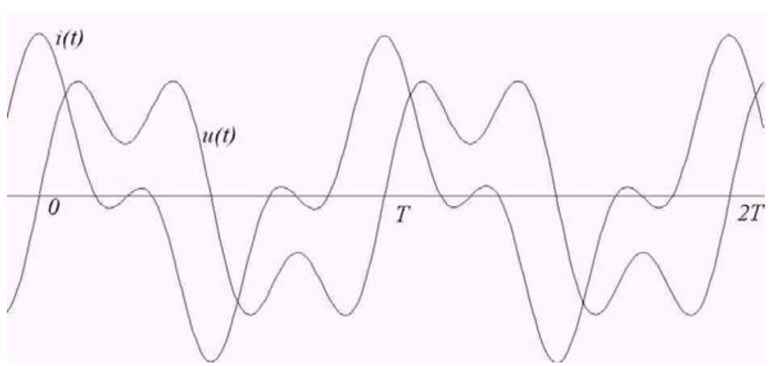
$$\mathbf{Y}_r = \mathbf{Y}_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$u(t) = \sqrt{2}(100 \sin \omega_1 t + 50 \sin 3\omega_1 t) \text{ V}$$




$$\mathbf{Y}_1 = \mathbf{Y}_3 = 1\, e^{j\frac{\pi}{2}} \text{ S}$$

$$i(t) = \sqrt{2}\, [100 \sin(\omega_1 t + \frac{\pi}{2}) + 50 \sin(3\omega_1 t + \frac{\pi}{2})] \text{ A}$$




**The load current is distorted in spite of zero distortion power , *D***

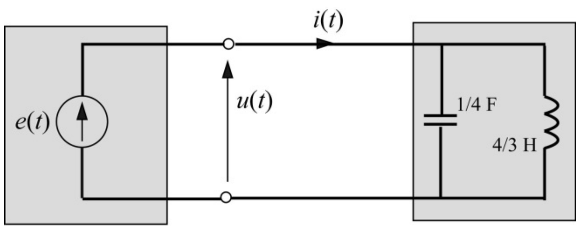
## The load current is not distorted, meaning

$$i(t) = a\, u(t-\tau)$$

$$\text{if} \qquad \boldsymbol{I}_n = a\,\boldsymbol{U}_n\, e^{-jn\tau} = \boldsymbol{Y}_n\, \boldsymbol{U}_n,$$

$$\boldsymbol{Y}_n = a\, e^{-jn\tau} \text{ ..........................................(2)}$$

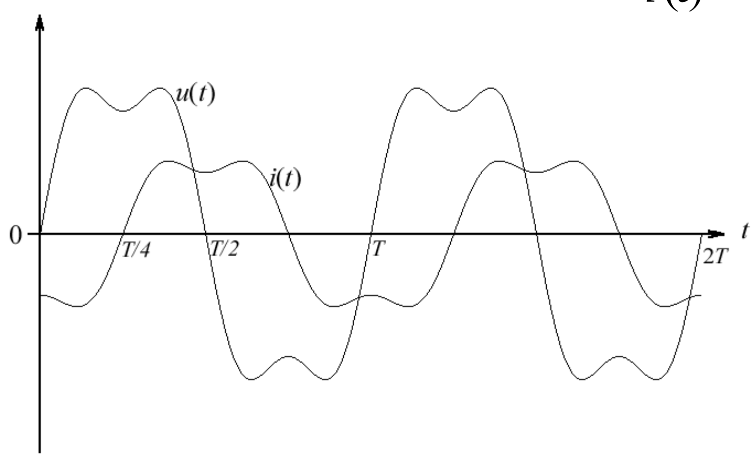
$$u(t) = \sqrt{2}(100\sin\omega_1 t + 30\sin 3\omega_1 t)\ \text{V}$$




$$\boldsymbol{Y}_1 = j\frac{1}{4} - j\frac{3}{4} = \frac{1}{2}e^{-j\frac{\pi}{2}}\ \text{S}$$

$$\boldsymbol{Y}_3 = j\frac{3}{4} - j\frac{1}{4} = \frac{1}{2}e^{j\frac{\pi}{2}} = \frac{1}{2}e^{-j3\frac{\pi}{2}}\ \text{S}$$

$$i(t) = \sqrt{2}[50\sin(\omega_1 t - \frac{\pi}{2}) + 15\sin(3\omega_1 t - \frac{3\pi}{2}) = \frac{1}{2}u(t - \frac{T}{4}),$$

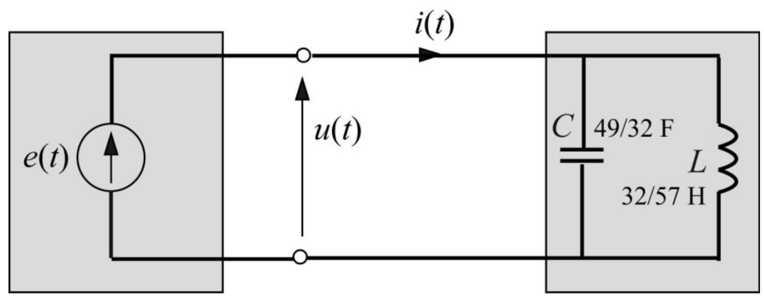



$$D = U_1 U_3 |\boldsymbol{Y}_1 - \boldsymbol{Y}_3| = 100\cdot 25\cdot |\frac{1}{2}e^{-j\frac{\pi}{2}} - \frac{1}{2}e^{j3\frac{\pi}{2}}| = 2.5\ \text{kVA}$$

**Load current is not distorted in spite of non zero distortion power, *D***

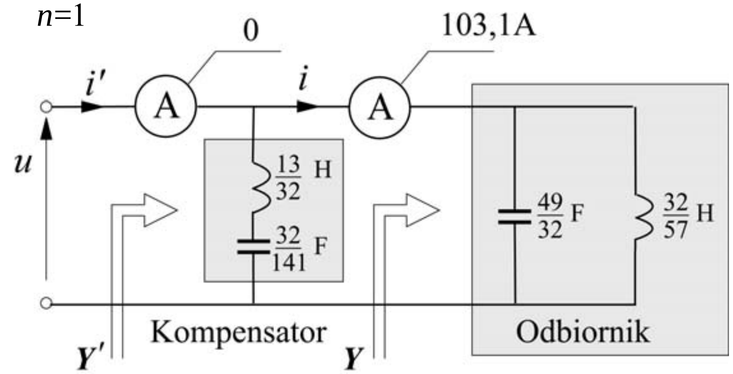
# Power factor improvement and Budeanu's reactive power

$$\| i \| = \sqrt{\sum_{n=0}^{N} \| i_n \|^2} = \sqrt{\sum_{n=0}^{N} (\frac{P_n}{U_n})^2 + \sum_{n=1}^{N} (\frac{Q_n}{U_n})^2}, \qquad \text{but in Budeanu Theory: } Q = \sum_{n=1}^{N} Q_n$$




$$u(t) = \sqrt{2}(100 \sin\omega_1 t + 25 \sin 3\omega_1 t)\ \mathrm{V} \qquad i(t) = \sqrt{2}\,[25 \sin(\omega_1 t - 90^0) + 100 \sin(3\omega_1 t + 90^0)]\ \mathrm{A}$$

$$Q = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n = 100\times 25\times 1 + 00\times 25\times(\text{-}1) = 0$$




**Budeanu's reactive power is useless for compensator design**

1931

S. Fryze, Professor of Lwow University, Poland, defined the reactive power in a time-domain, based on

the load current orthogonal decomposition into active and reactive currents

$$i = i_{\mathrm{a}} + i_{\mathrm{rF}}$$

$$i_{\mathrm{a}}(t) \overset{\mathrm{df}}{=} \frac{P}{\|u\|^2} u(t) \overset{\mathrm{df}}{=} G_{\mathrm{e}}\, u(t), \qquad i_{\mathrm{rF}}(t) \overset{\mathrm{df}}{=} i(t) - i_{\mathrm{a}}(t)$$

$$\frac{1}{T}\int_0^T i_{\mathrm{a}}(t)\, i_{\mathrm{rF}}(t)\, dt = (i_{\mathrm{a}}, i_{\mathrm{rF}}) = 0$$

$$\|i\|^2 = \|i_{\mathrm{a}}\|^2 + \|i_{\mathrm{rF}}\|^2$$

Fryze's Power Equation: $S^2 = P^2 + Q_{\mathrm{F}}^2$

Fryze’s definition of reactive power: $Q_{\mathrm{F}} \overset{\mathrm{df}}{=} \|u\|\,\|i_{\mathrm{rF}}\|$

1997

L.S. Czarnecki: „Budeanu and Fryze: Two frameworks for Interpreting Power Properties of Circuits with Nonsinusoidal Voltages and Currents,” *Archiv fur Elektrotechnik*

Fryze's Power Equation

$$S^2 = P^2 + Q_F^2$$

$$i = i_a + i_{rF}$$

$i_a$ - Active current
$i_{rF}$ - Reactive current

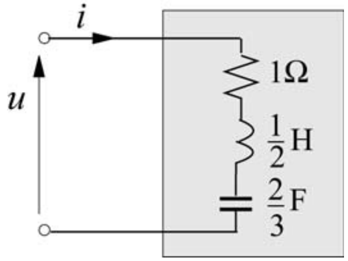
We know that the following phenomena may contribute to the power factor deterioration

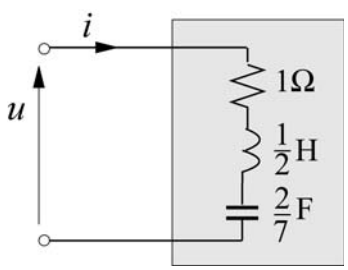
1. Energy oscillations
2. Bi-directional flow of active power
3. Harmonic generation in the load
4. Change of the load conductance with frequency
5. Load unbalance

Fryze's Power Theory does not explain the effect of these power phenomena on the power factor

Question: Does the Fryze's Power Theory provide fundamentals for the power factor improvement?

$$u(t) = 100\sqrt{2}\,(\sin\omega_1 t + \sin 3\omega_1 t)\ \ \mathrm{V}$$



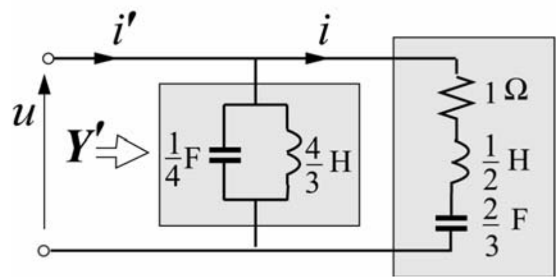




$P$ = 10 kW

$Q$ = 10 kVAr

$S$ = 14.1 kVA

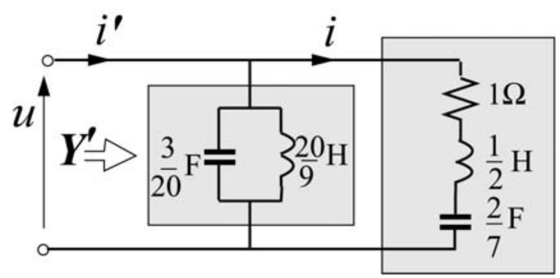
$\lambda$ = 0.71




Q = 0

$S$ = 10 kVA

$\lambda$ = 1




Q = 8 kVAr

$S$ = 12.7 kVA

$\lambda$ = 0.78

These loads cannot be distinguished with respect to Fryze's powers.

They differ as to the possibility of their compensation

Fryze's Power Theory

does not enable us to draw conclusions as to the possibility of the load compensation with a reactive compensator

# Opinion: Fryze's power theory provides fundamentals for switching compensator control

$$i = i_a + i_{rF}$$

$i_a$ - active current is useful component
$i_{rF}$ - reactive current is useless component

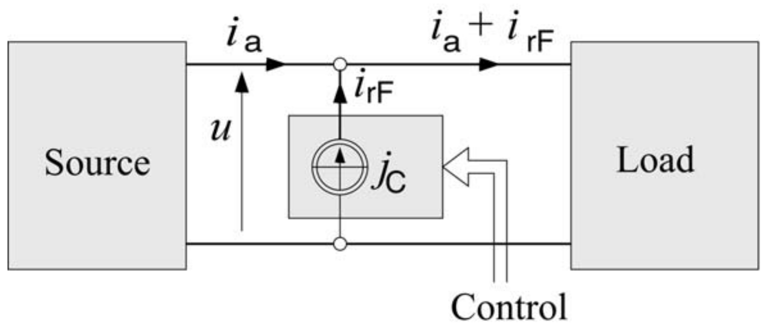
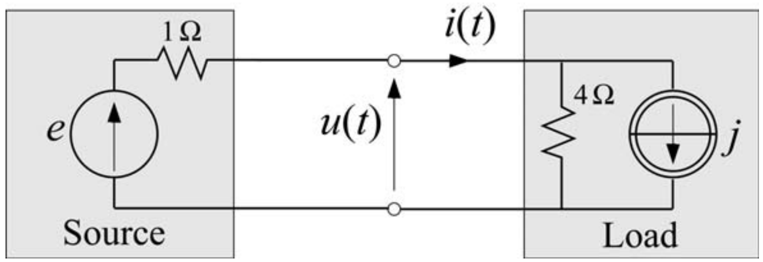


**Illustration:**

$e = 100\sqrt{2}\,\sin\omega_1 t$ V




$j = 50\sqrt{2}\,\sin 3\omega_1 t$ A

$$i = \sqrt{2}(20\,\sin\omega_1 t + 40\,\sin 3\omega_1 t)\ \mathrm{A}$$

$$u = \sqrt{2}(80\,\sin\omega_1 t - 40\,\sin 3\omega_1 t)\ \mathrm{V}$$

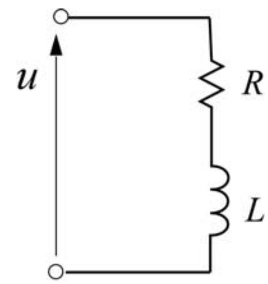
$$P = 1600 - 1600 = 0 \qquad i_\mathrm{a}(t) = \frac{P}{\|u\|^2}u(t) = 0$$

According to Fryze's Power Theory,
total compensation requires that the current $i_{\mathrm{rF}}$ is reduced to zero

This is a wrong conclusion

Only the 3rd order current harmonic should be compensated

$$u(t) = U_0 + \sqrt{2}\sum_{n=1}^{\infty} U_n \cos(n\omega_1 t + \alpha_n)$$




1927: Budeanu: $S^2 = P^2 + Q_B^2 + D^2$ $\quad Q_B \overset{\mathrm{df}}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$

1931: Fryze: $S^2 = P^2 + Q_F^2$ $\quad Q_F \overset{\mathrm{df}}{=} \|u\|\|i_{rF}\|$

1971: Shepherd: $S^2 = S_R^2 + Q_S^2$ $\quad Q_S \overset{\mathrm{df}}{=} \|u\|\|i_{rS}\|$

1975: Kusters: $S^2 = P^2 + Q_K^2 + Q_r^2$ $\quad Q_K \overset{\mathrm{df}}{=} \|u\|\|i_{rC}\|$

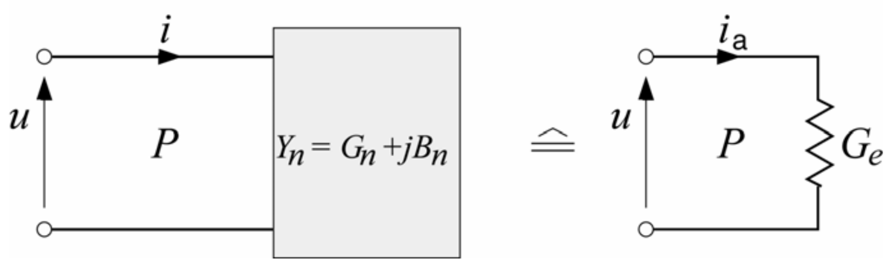
1979: Depenbrock: $S^2 = P^2 + Q_1^2 + V^2 + N^2$

1984 r. Czarnecki: $S^2 = P^2 + Q^2 + D_s^2$

# Current Physical Components (CPC) Power Theory
# of Linear Single-Phase Circuits
# with Nonsinusoidal Voltages and Currents

$$u = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

$$i = G_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \boldsymbol{Y}_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$




$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}}$$

$$i_{\mathrm{a}} = G_{\mathrm{e}}\, u, \qquad G_{\mathrm{e}} = \frac{P}{\|u\|^2}$$

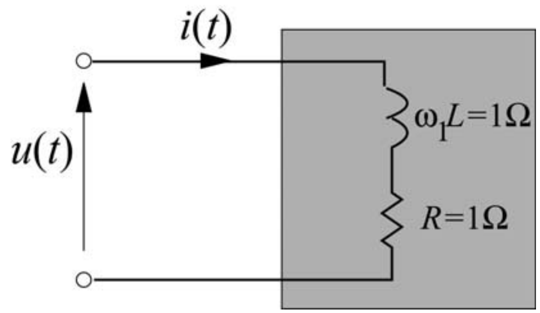
**Active current**

$$i_{\mathrm{s}} = (G_0 - G_{\mathrm{e}})U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} (G_n - G_{\mathrm{e}})\boldsymbol{U}_n\, e^{jn\omega_1 t}$$

**Scattered current**

$$i_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \boldsymbol{U}_n\, e^{jn\omega_1 t}$$

**Reactive current**




$$G_n = \mathrm{Re}\{\boldsymbol{Y}_n\} = \mathrm{Re}\frac{1}{R + jn\omega_1 L} = \frac{R}{R^2 + (n\omega_1 L)^2}$$

$$G_0 = 1\ \mathrm{S},\quad G_1 = 0{,}5\ \mathrm{S},\quad G_2 = 0{,}2\ \mathrm{S},\quad G_3 = 0{,}1\ \mathrm{S},\quad G_4 = 0{,}06\ \mathrm{S}.$$

Currents $i_a$, $i_s$ and $i_r$ are mutually orthogonal

$$i = i_a + i_s + i_r$$

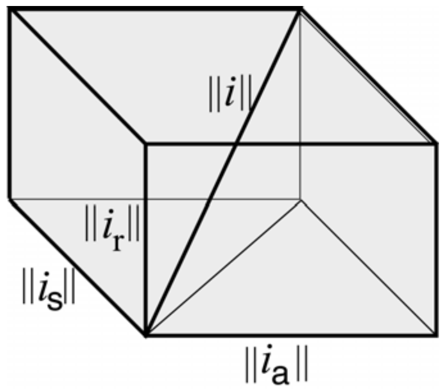
thus

$$||i||^2 = ||i_a||^2 + ||i_s||^2 + ||i_r||^2$$

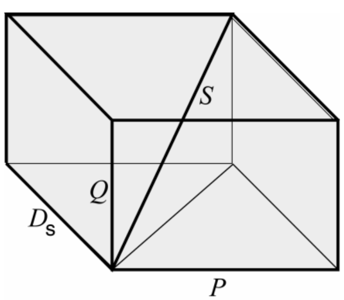
$$||i_a|| = G_e ||u||$$

$$||i_s|| = \sqrt{\sum_{n=0}^{\infty} (G_n - G_e)^2 U_n^2}$$

$$||i_r|| = \sqrt{\sum_{n=1}^{\infty} B_n^2 U_n^2}$$




Multiplying the current RMS equation by $||u||^2$

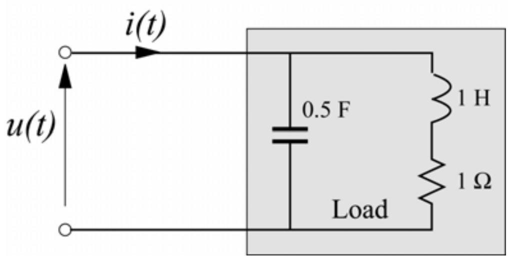



$$S^2 = P^2 + D_s^2 + Q^2$$

This decomposition and power equation
was developed without any approximation consequently,
this decomposition is valid independently on the level of harmonic distortion

## *Illustration*

$$u(t) = 50 + \sqrt{2}\,\mathrm{Re}\{100 e^{j\omega_1 t} + 20 e^{j5\omega_1 t}\}\ \mathrm{V},\quad \omega_1 = 1\ \mathrm{rd/s} \qquad \|u\| = 113.58\ \mathrm{V}$$




$Y_0 = 1$ S

$Y_1 = 0.5$ S

$Y_5 = 0.04 + j2.31$ S

$$P = \sum_{n=0,1,5} G_n U_n^2 = 7516\ \mathrm{W}$$

$$i(t) = 50 + \sqrt{2}\,\mathrm{Re}\{50 e^{j\omega_1 t} + 46.2 e^{j89^0} e^{j5\omega_1 t}\}\ \mathrm{A},\qquad \|i\| = \sqrt{50^2 + 50^2 + 46.2^2} = 84.47\ \mathrm{A}$$

$$G_\mathrm{e} = \frac{P}{\|u\|^2} = 0.5826\ \mathrm{S}$$

$$\|i_\mathrm{a}\| = G_\mathrm{e}\|u\| = 66.17\ \mathrm{A}$$

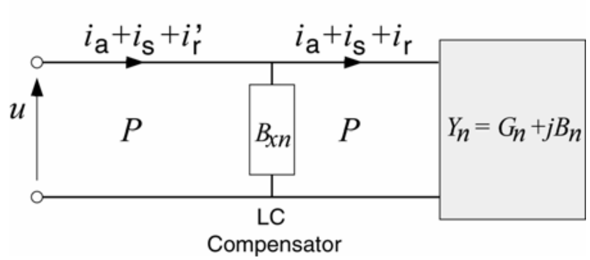
$$\|i_\mathrm{s}\| = \sqrt{\sum_{n=0,1,5} (G_n - G_\mathrm{e})^2 U_n^2} = 24.93\ \mathrm{A}$$

$$\|i\| = \sqrt{\|i_\mathrm{a}\|^2 + \|i_\mathrm{s}\|^2 + \|i_\mathrm{r}\|^2} = 84.47\ \mathrm{A}$$

$$\|i_\mathrm{r}\| = \sqrt{\sum_{n=1,5} B_n^2 U_n^2} = 46.2\ \mathrm{A}$$

In real systems the scattered current has relatively small value

# Reactive current compensation:

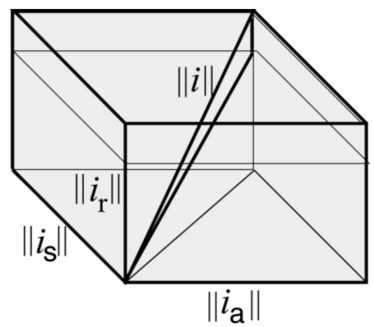



Lossless shunt reactive compensators do not change active power, $P$, and conductance $G_n$.

$$G_{\mathrm{e}} = \frac{P}{\|u\|^2} = \text{const.}$$

$$\|i_{\mathrm{a}}\| = G_{\mathrm{e}}\|u\| = \text{const.}$$

$$\|i_{\mathrm{s}}\| = \sqrt{\sum_{n=0}^{\infty}(G_n - G_{\mathrm{e}})^2 U_n^2} = \text{const.}$$




The RMS value of the reactive current changes to: $\|i_{\mathrm{r}}'\| = \sqrt{\sum_{n=1}^{\infty}(B_n + B_{xn})^2 U_n^2}$

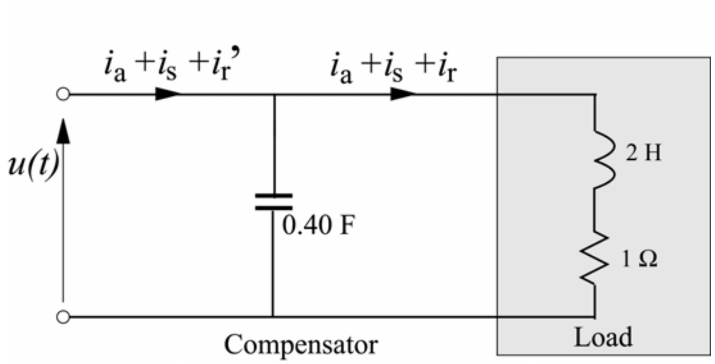
A total compensation of the reactive current:

$$\|i_{\mathrm{r}}'\| = 0, \text{ if for each } n, \text{ such that } U_n \neq 0, \; B_{\mathrm{x}\,n} = -B_n$$

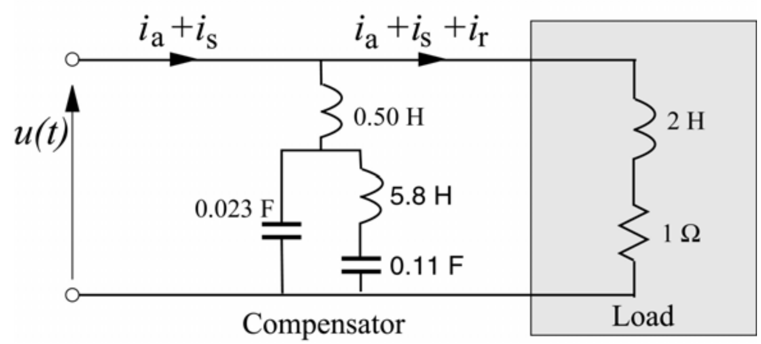
Conclusion: This decomposition solves the problem of a shunt reactive compensation of linear loads

## *Illustration*

$$u(t) = \sqrt{2}\,\mathrm{Re}\{100e^{j\omega_1 t} + 5e^{j5\omega_1 t}\}\ \mathrm{V} \qquad \omega_1 = 1\ \mathrm{rad/s}$$




$Y_1 = 0.20 - j0.40$ S
$Y_5 = 0.01 - j0.10$ S




$Y_1' = 0.20$ S
$Y_5' = 0.01 + j1.9$ S

$Y_1' = 0.20$ S
$Y_5' = 0.01$ S

$$i(t) = \sqrt{2}\,\mathrm{Re}\{20e^{j\omega_1 t} + 9.5e^{j89^\circ}e^{j5\omega_1 t}\}\ \mathrm{A}$$

$$\|i_a\| = 19.98\ \mathrm{A}$$
$$\|i_s\| = 0.95\ \mathrm{A}$$
$$\|i_r'\| = 9.50\ \mathrm{A}$$

$$i(t) = \sqrt{2}\,\mathrm{Re}\{20e^{j\omega_1 t} + 0.05\,e^{j5\omega_1 t}\}\ \mathrm{A}$$

$$\|i_a\| = 19.98\ \mathrm{A}$$
$$\|i_s\| = 0.95\ \mathrm{A}$$
$$\|i_r'\| = 0$$

## Power factor

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + D_s^2 + Q^2}} = \frac{\|i_a\|}{\sqrt{\|i_a\|^2 + \|i_s\|^2 + \|i_r\|^2}}$$

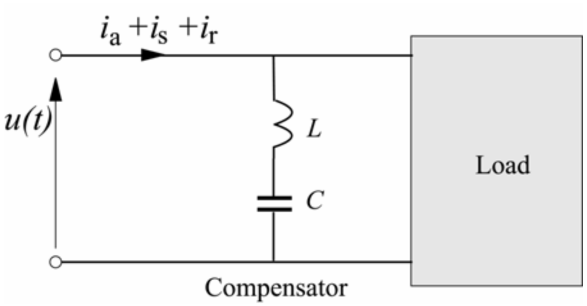
$$\lambda = 0.867$$

$$\lambda = 0.999$$

# Reactive current minimization

Total compensation requires very complex compensators, therefore, it has only a theoretical value.

The reactive current can be minimized by two-element reactive compensator




$$B_{xn} = \frac{n\omega_1 C}{1 - n^2\omega_1^2 LC}$$

$$||i_r|| = \text{Min., if } \frac{d}{dC}||i_r|| = 0$$

>>

$$C_{k+1} = -\frac{\sum_{n\in N} \frac{nB_n U_n^2}{(1-n^2\omega_1^2 LC_k)^2}}{\omega_1 \sum_{n\in N} \frac{n^2 B_n U_n^2}{(1-n^2\omega_1^2 LC_k)^3}} \rightarrow C_{\text{opt}}$$

The inductance *L* can be chosen such that no resonance occur. The circuit can operate at close to unity power factor

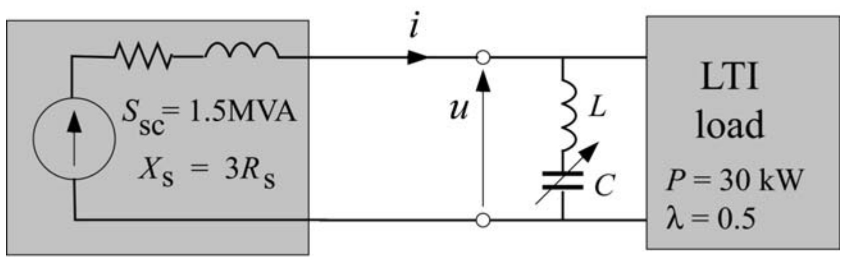
# Illustration

$E_3 = 1\%\ E_1$

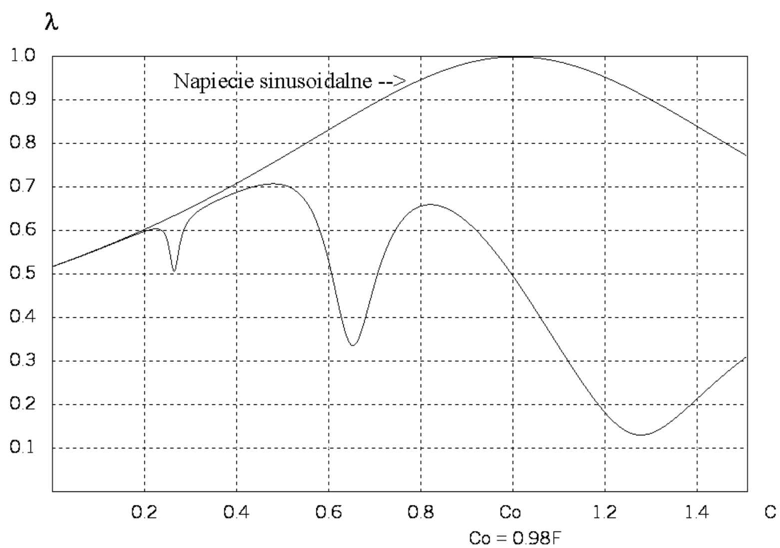
$E_5 = 5\%\ E_1$

$E_7 = 2\%\ E_1$

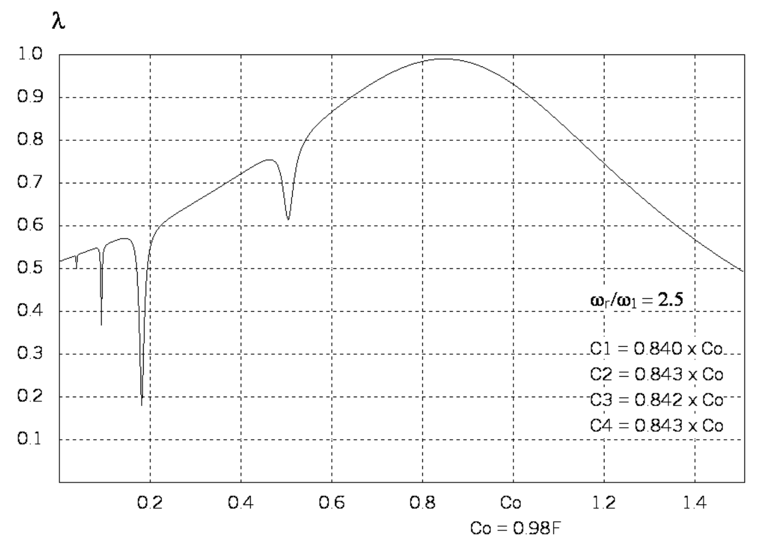
$E_{11} = 1\%\ E_1$




$L = 0$



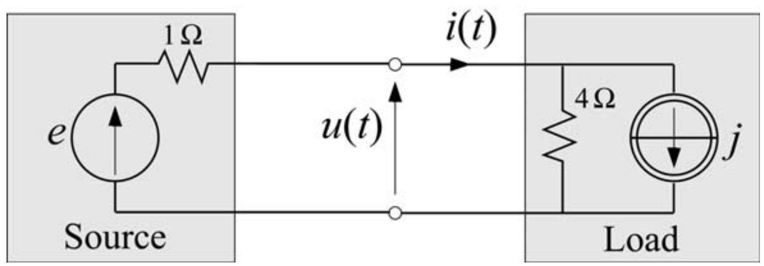

$L$ such that $\omega_r = 2.5\,\omega_1$




$$C_{k+1} = -\frac{\displaystyle\sum_{n\in N} \frac{n B_n U_n^2}{(1-n^2\omega_1^2 L C_k)^2}}{\displaystyle\omega_1 \sum_{n\in N} \frac{n^2 B_n U_n^2}{(1-n^2\omega_1^2 L C_k)^3}} \rightarrow C_{opt} = 0.85\, C_0 \quad \text{in three steps of iteration}$$

## Moce w owodach z odbiornikami generującymi harmoniczne (HGL)

$e = 100\sqrt{2}\,\sin\omega_1 t$ V




$j = 50\sqrt{2}\,\sin 3\omega_1 t$ A

$$i = \sqrt{2}(20\sin\omega_1 t + 40\sin 3\omega_1 t)\ \mathrm{A}$$

$$u = \sqrt{2}(80\sin\omega_1 t - 40\sin 3\omega_1 t)\ \mathrm{V}$$

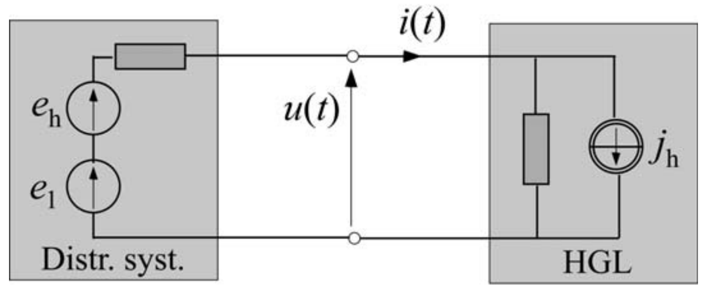
$\|i\| = 44.72$ A, $\|u\| = 89.44$ V, $S = 4000$ VA

$P = 1600 - 1600 = 0,\quad Q = 0,\quad D_s = 0,\quad$ lecz $\quad S \neq 0$

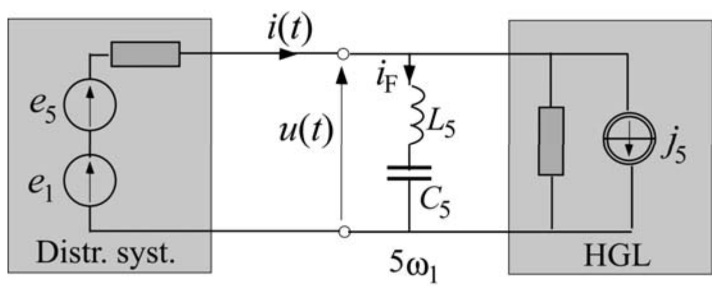
Równanie:

$$S^2 = P^2 + D_s^2 + Q^2$$

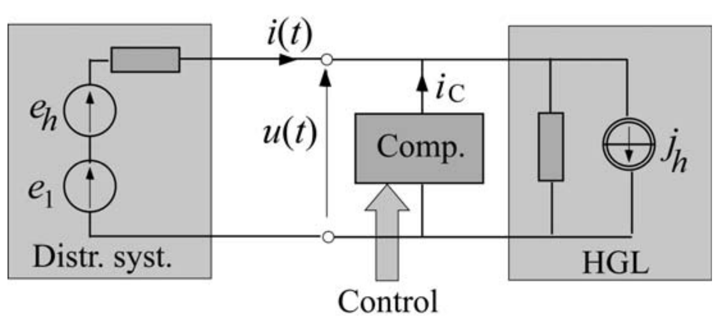
**nie jest poprawne!!!!**

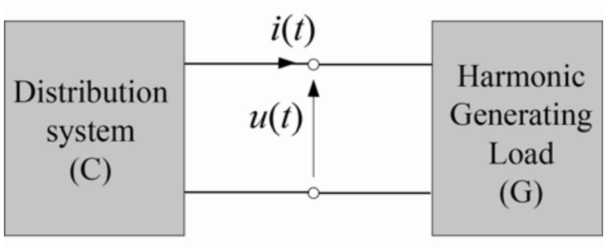



Lokalizacja źródeł harmonicznych ma krytyczne znaczenie dla projektowania filtrów harmonicznych




i kompensatorów harmonicznych






$$u = \sum_{n\in N} u_n, \quad i = \sum_{n\in N} i_n, \quad P = \sum_{n\in N} P_n$$

$$P_n = U_n\, I_n \cos\varphi_n \begin{cases} \geq 0 \\ < 0 \end{cases}$$

Ze wzgledu na kierunek przepływu energii harmonicznych
zbiór rzędów harmonicznych $N$ może być rozłożony na dwa podzbiory
$N_C$, and $N_G$,

$$\text{Jesli } |\varphi_n| \leq 90^0, \quad n \in N_C$$

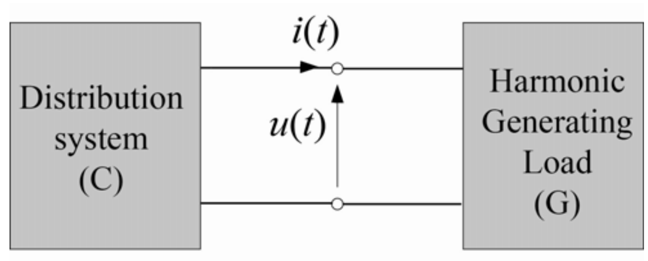
$$\text{Jesli } |\varphi_n| > 90^0, \quad n \in N_G$$

$$\sum_{n\in N_C} u_n \overset{\text{df}}{=} u_C, \quad \sum_{n\in N_C} i_n \overset{\text{df}}{=} i_C, \quad \sum_{n\in N_C} P_n \overset{\text{df}}{=} P_C$$

$$\sum_{n\in N_G} u_n \overset{\text{df}}{=} -u_G, \quad \sum_{n\in N_G} i_n \overset{\text{df}}{=} i_G, \quad \sum_{n\in N_G} P_n \overset{\text{df}}{=} -P_G$$

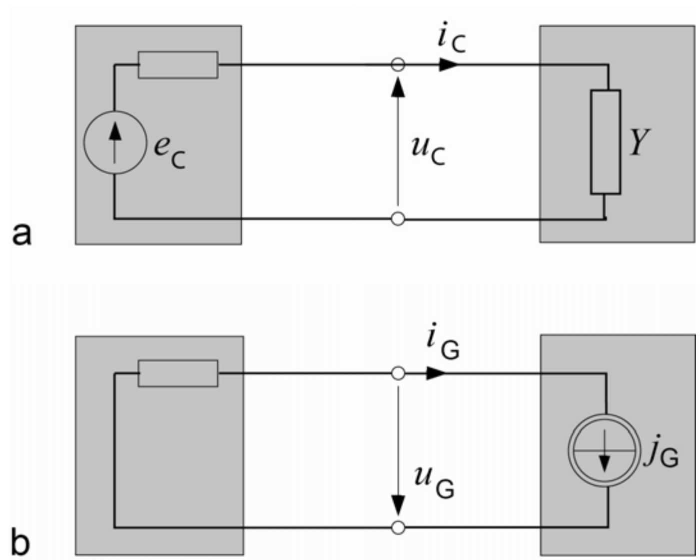
$$u = u_C - u_G, \quad i = i_C + i_G, \quad P = P_C - P_G$$

$$\|u\|^2 = \|u_C\|^2 + \|u_G\|^2 \qquad \|i\|^2 = \|i_C\|^2 + \|i_G\|^2$$




## Obwody równoważne:

Dla $n \in N_C$



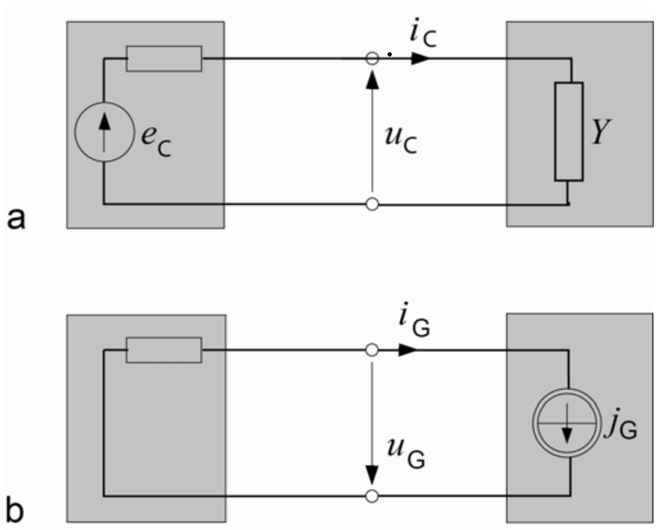

$$\boldsymbol{Y}_n \overset{\mathrm{df}}{=} G_n + jB_n = \frac{\boldsymbol{I}_n}{\boldsymbol{U}_n}$$

Dla $n \in N_G$

$$i = i_C + i_G$$

# Rozkład prądu według CPC:

Dla $n \in N_C$



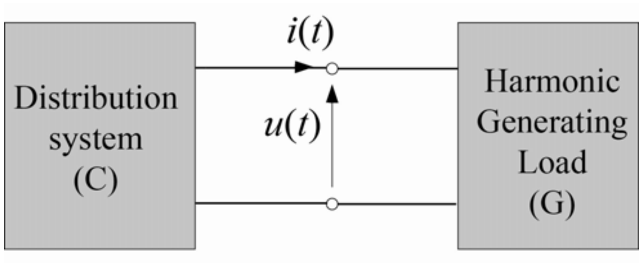

$$\boldsymbol{Y}_n \overset{\text{df}}{=} G_n + jB_n = \frac{\boldsymbol{I}_n}{\boldsymbol{U}_n}$$

Dla $n \in N_G$

$$G_{\text{eC}} \overset{\text{df}}{=} \frac{P_{\text{C}}}{\|u_{\text{C}}\|^2} \qquad i_{\text{aC}} \overset{\text{df}}{=} G_{\text{eC}}\, u_{\text{C}} \qquad i = i_{\text{aC}} + i_{\text{sC}} + i_{\text{rC}} + i_{\text{G}}$$

# Rozkład prądu według Fryzego:

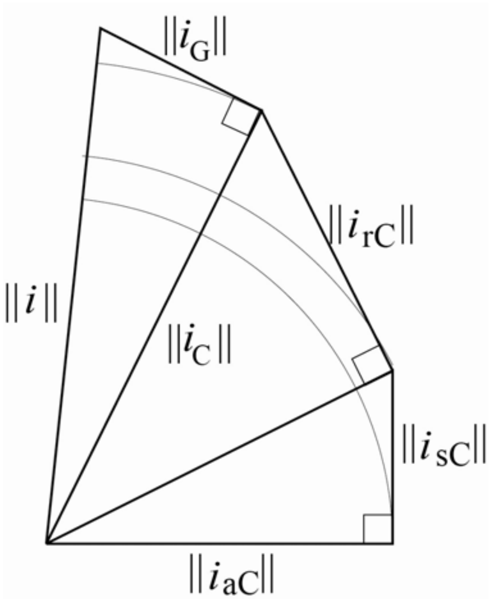
$$G_{\text{eF}} \overset{\text{df}}{=} \frac{P}{\|u\|^2} \qquad i_{\text{aF}} \overset{\text{df}}{=} G_{\text{eF}}\, u \qquad i = i_{\text{aF}} + i_{\text{rF}},$$




Składowe fizyczne prądu odbiornika generującego harmoniczne:

$$i = i_{aC} + i_{sC} + i_{rC} + i_{G}$$

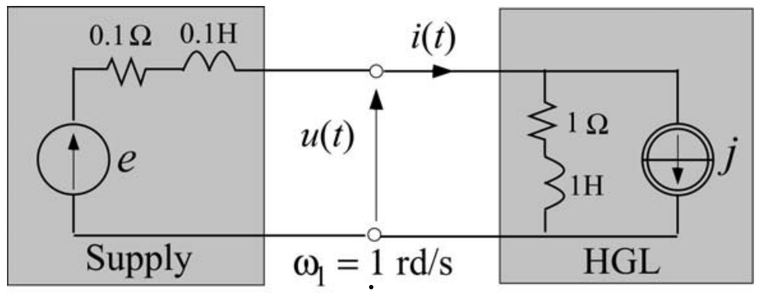
to jest stwarzyszone z odrębnymi zjawiskami fizycznymi




Prądy te są wzajemnie ortogonalne

$$\| i \|^2 = \| i_{aC} \|^2 + \| i_{sC} \|^2 + \| i_{rC} \|^2 + \| i_{G} \|^2$$

## Przykład




$$e = 220\sqrt{2}\cos\omega_1 t + 15\sqrt{2}\cos 3\omega_1 t \text{ V}$$

$$j = 20\sqrt{2}\cos 5\omega_1 t + 15\sqrt{2}\cos 7\omega_1 t \text{ A}$$

$N_C = \{1,3\}$

$\boldsymbol{U}_1 = 200.0 \text{ V}, \quad \boldsymbol{I}_1 = 141.42\, e^{-j45^0} \text{ A} \quad Y_1 = 0.50 - j0.50 \text{ S}$

$\boldsymbol{U}_3 = 13.64\, e^{j8.4^0} \text{ V}, \quad \boldsymbol{I}_3 = 4.31\, e^{-j71.6^0} \text{ A} \quad Y_3 = 0.10 - j0.30$

$N_G = \{5,7\}$

$\boldsymbol{U}_5 = 9.27\, e^{-j101.3^0} \text{ V}, \quad \boldsymbol{I}_5 = 18.18 \text{ A}$

$\boldsymbol{U}_7 = 9.64\, e^{-j98.1^0} \text{ V}, \quad \boldsymbol{I}_7 = 13.64 \text{ A}$

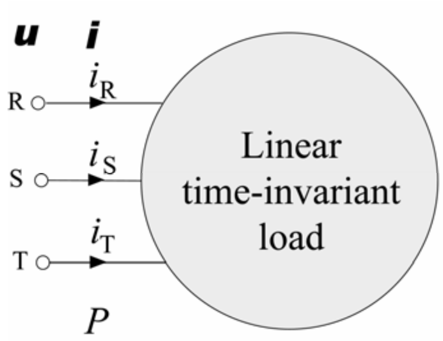
$$P_C = \mathrm{Re}\{\boldsymbol{U}_1 \boldsymbol{I}_1^*\} + \mathrm{Re}\{\boldsymbol{U}_3 \boldsymbol{I}_3^*\} = 20010 \text{ W}, \quad \|u_C\| = \sqrt{U_1^2 + U_3^2} = 200.46 \text{ V}, \quad G_{eC} = \frac{P_C}{\|u_C\|^2} = 0.4979 \text{ S}$$

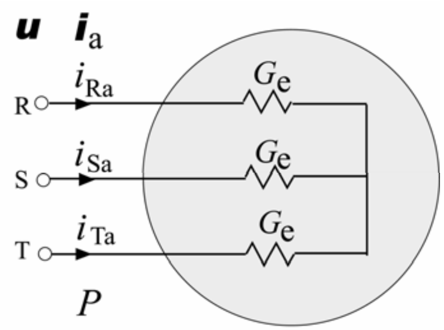
$$\|i_{aC}\| = \frac{P_C}{\|u_C\|} = 99.84 \text{ A}, \quad \|i_{sC}\| = \sqrt{\sum_{n=1,3} [(G_n - G_{eC})\, U_n]^2} = 5.44 \text{ A}, \quad \|i_{rC}\| = \sqrt{\sum_{n=1,3} (B_n U_n)^2} = 100.08 \text{ A}$$

$$\|i_G\| = \sqrt{\sum_{n=5,7} I_n^2} = 22.73 \text{ A}$$

# Składowe fizyczne prądu odbiornika LTI zasilanego w trójprzewodowo symetrycznym niesinusoidalnym napięciem

$$\boldsymbol{u} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} U_n\, e^{jn\omega_1 t}$$




$$G_e \stackrel{\mathrm{df}}{=} \frac{P}{\|\boldsymbol{u}\|^2}$$

Active current:

$$\boldsymbol{i}_a \stackrel{\mathrm{df}}{=} G_e\,\boldsymbol{u}$$

Składowa bezużyteczna prądu:

$$\boldsymbol{i} - \boldsymbol{i}_a = \sum_{n\in N} \boldsymbol{i}_n - \boldsymbol{i}_a = \sum_{n\in N} (\boldsymbol{i}_{an} + \boldsymbol{i}_{rn} + \boldsymbol{i}_{un}) - \boldsymbol{i}_a = (\sum_{n\in N} \boldsymbol{i}_{an} - \boldsymbol{i}_a) + \sum_{n\in N} \boldsymbol{i}_{rn} + \sum_{n\in N} \boldsymbol{i}_{un}$$

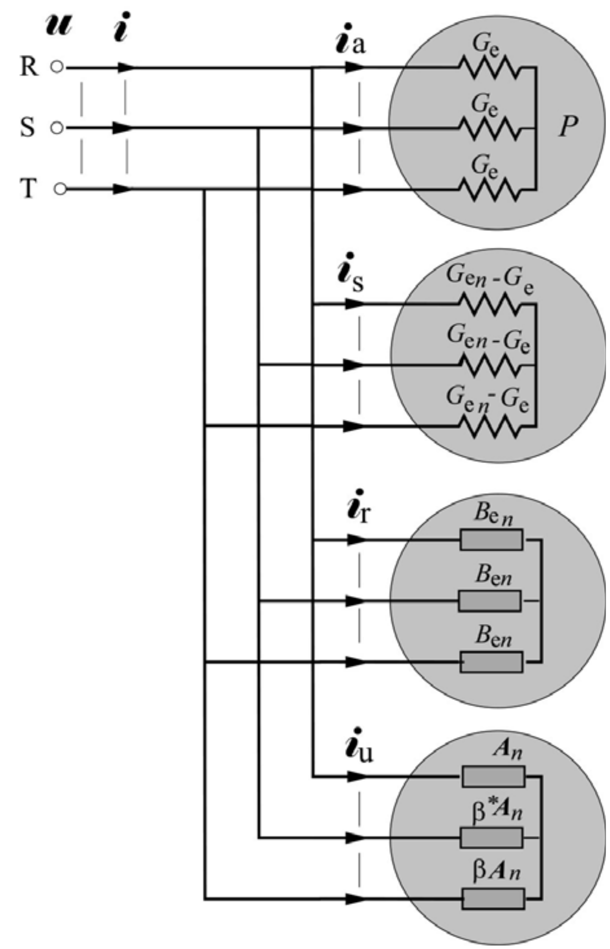
## Składowa bezużyteczna prądu:

$$\boldsymbol{i}-\boldsymbol{i}_{\mathrm{a}}=\sum_{n\in N}\boldsymbol{i}_n-\boldsymbol{i}_{\mathrm{a}}=\sum_{n\in N}(\boldsymbol{i}_{\mathrm{a}n}+\boldsymbol{i}_{\mathrm{r}n}+\boldsymbol{i}_{\mathrm{u}n})-\boldsymbol{i}_{\mathrm{a}}=(\sum_{n\in N}\boldsymbol{i}_{\mathrm{a}n}-\boldsymbol{i}_{\mathrm{a}})+\sum_{n\in N}\boldsymbol{i}_{\mathrm{r}n}+\sum_{n\in N}\boldsymbol{i}_{\mathrm{u}n}$$

$$\boldsymbol{i}_{\mathrm{s}}\stackrel{\mathrm{df}}{=}\sum_{n\in N}\boldsymbol{i}_{\mathrm{a}n}-\boldsymbol{i}_{\mathrm{a}}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}(G_{\mathrm{e}n}-G_{\mathrm{e}})\,U_n\,e^{jn\omega_1 t}$$

$$\boldsymbol{i}_{\mathrm{r}}\stackrel{\mathrm{df}}{=}\sum_{n\in N}\boldsymbol{i}_{\mathrm{r}n}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}jB_{\mathrm{e}n}\,U_n\,e$$

$$\boldsymbol{i}_{\mathrm{u}}\stackrel{\mathrm{df}}{=}\sum_{n\in N}\boldsymbol{i}_{\mathrm{u}n}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}A_n\,U_n^{\#}\,e^{jn\omega_1 t}$$




## Rozkład na składowe fizyczne

$$\boldsymbol{i}=\boldsymbol{i}_{\mathrm{a}}+\boldsymbol{i}_{\mathrm{s}}+\boldsymbol{i}_{\mathrm{r}}+\boldsymbol{i}_{\mathrm{u}}$$

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}$$

Wartości skuteczne składowych fizycznych:

$$\|\boldsymbol{i}_{\mathrm{a}}\| = G_{\mathrm{e}}\,\|\boldsymbol{u}\|,$$

$$\|\boldsymbol{i}_{\mathrm{s}}\| = \sqrt{\sum_{n\in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})^2\, \|\boldsymbol{u}_n\|^2}$$

$$\|\boldsymbol{i}_{\mathrm{u}}\| = \sqrt{\sum_{n\in N} A_n\, \|\boldsymbol{u}_n\|^2}$$

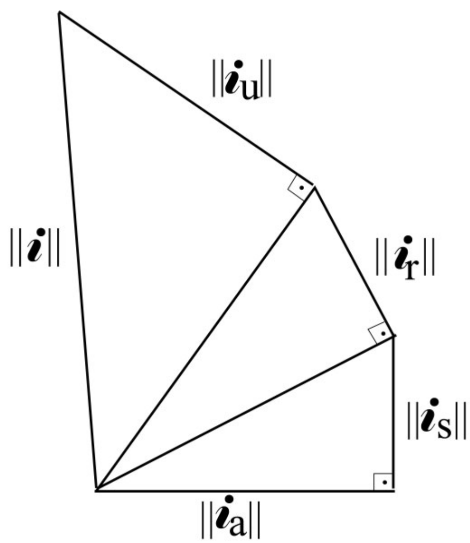
$$\|\boldsymbol{i}_{\mathrm{r}}\| = \sqrt{\sum_{n\in N} |\boldsymbol{B}_n|^2\, \|\boldsymbol{u}_n\|^2}$$

Prąd rozrzutu pojawia się wtedy, gdy

$$G_{\mathrm{e}} = \frac{P}{\|\boldsymbol{u}\|^2}, \qquad G_{\mathrm{e}n} = \frac{P_n}{\|\boldsymbol{u}_n\|^2}$$

CPS są wzajemnie ortogonalne, zatem:

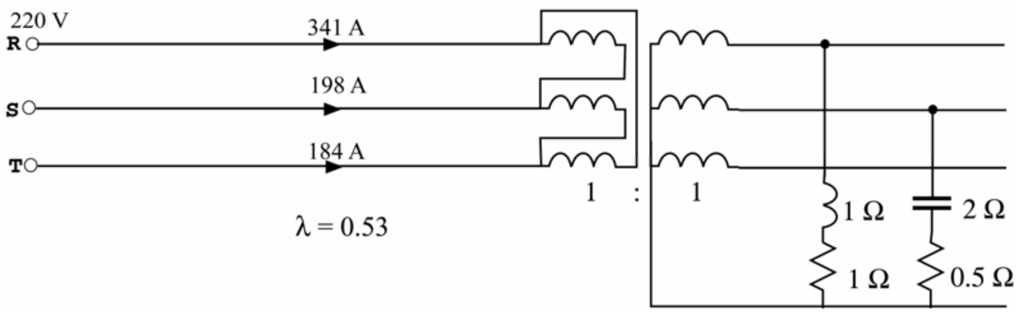
$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{s}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}\|^2$$



# Przykład

Rozkład na składowe symetryczne nie zależy od poziomu odkształcenia

$$u_R(t) = \sqrt{2}\,\mathrm{Re}\{220e^{j\omega_1 t} + 44e^{j5\omega_1 t}\}\ \mathrm{V}$$




$G_e = 0.6018$ S

$Y_{e1} = 0.60 - j0.40$ S

$Y_{e5} = 0.88 + j0.15$ S

$A_1 = 0.83\ e^{-j0.18p}$ S

$A_5 = 1.12\ e^{-j0.86p}$ S

$$||\boldsymbol{i}|| = \sqrt{||i_R||^2 + ||i_S||^2 + ||i_T||^2} = \sqrt{341^2 + 198^2 + 184^2} = 433\ \mathrm{A}$$

$$||\boldsymbol{i}_a|| = 237\ \mathrm{A}$$

$$||\boldsymbol{i}_s|| = 21\ \mathrm{A}$$

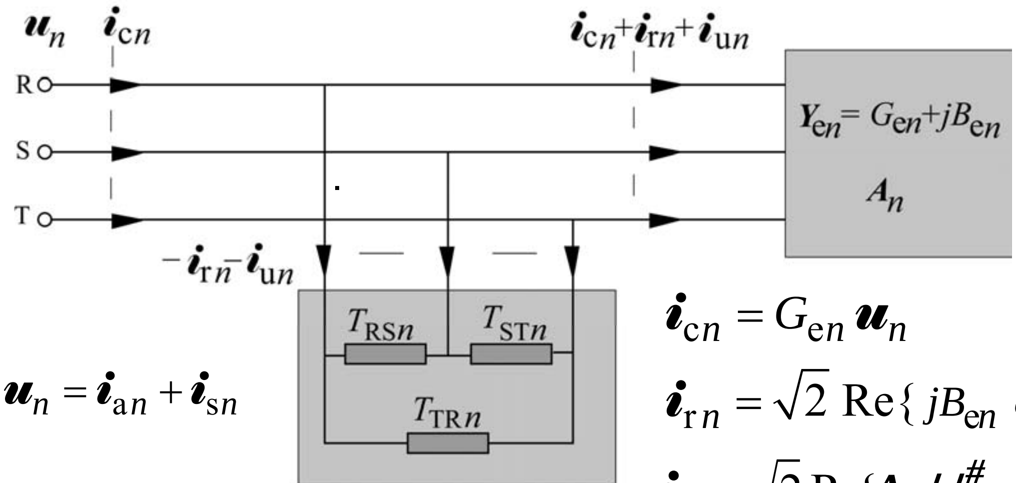
$$||\boldsymbol{i}_r|| = 153\ \mathrm{A}$$

$$||\boldsymbol{i}_u|| = 327\ \mathrm{A}$$

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_s||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2} = \sqrt{237^2 + 21^2 + 153^2 + 327^2} = 433\ \mathrm{A}$$

# Kompensacja reaktancyjna w warumkach niesinusoidalnych

Dla harmonicznej rzędu *n*:




$$\boldsymbol{i}_{cn} = G_{en}\,\boldsymbol{u}_n = G_e\,\boldsymbol{u}_n + (G_{en}-G_e)\,\boldsymbol{u}_n = \boldsymbol{i}_{an} + \boldsymbol{i}_{sn}$$

$$\boldsymbol{i}_{cn} = G_{en}\,\boldsymbol{u}_n$$

$$\boldsymbol{i}_{rn} = \sqrt{2}\,\mathrm{Re}\{\,jB_{en}\,\boldsymbol{U}_n\,e^{jn\omega_1 t}\}$$

$$\boldsymbol{i}_{un} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}_n\,\boldsymbol{U}_n^{\#}\,e^{jn\omega_1 t}\}$$

$$B_{en} + (T_{STn} + T_{TRn} + T_{RSn}) = 0$$

$$\boldsymbol{A}_n - j\,(T_{STn} + \beta\,T_{TRn} + \beta^{*}T_{RSn}) = 0$$

>>>

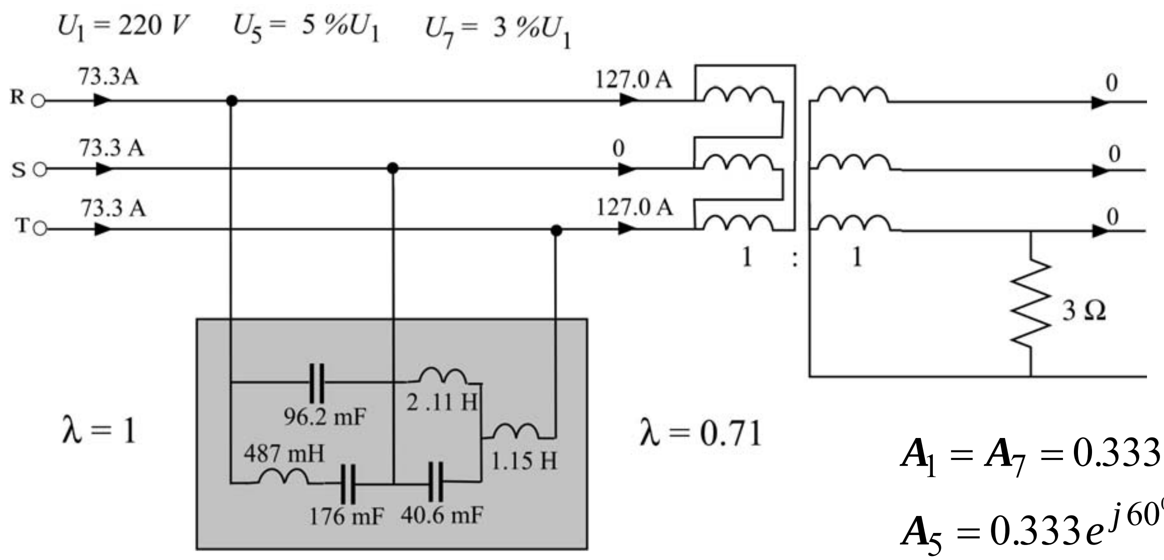
$$T_{RSn} = \frac{1}{3}(s\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}_n - \mathrm{Im}\,\boldsymbol{A}_n - B_{en})$$

$$T_{STn} = \frac{1}{3}(2\,\mathrm{Im}\,\boldsymbol{A}_n - B_{en})$$

$$T_{TRn} = -\frac{1}{3}(s\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A}_n + \mathrm{Im}\,\boldsymbol{A}_n + B_{en})$$

L.S. Czarnecki,
"Reactive and unbalanced currents compensation in three-phase circuits under nonsinusoidal conditions,"
*IEEE Trans. Instr. Measur.,* Vol. IM-38, No. 3, pp. 754-459, June 1989.

# Przykład




$$A_1 = A_7 = 0.333 e^{-j60^0}\ \mathrm{S}$$

$$A_5 = 0.333 e^{j60^0}\ \mathrm{S}$$

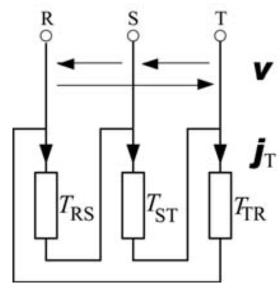
$$T_{\mathrm{RS}n} = \frac{1}{3}(s\sqrt{3}\,\mathrm{Re}\,A_n - \mathrm{Im}\,A_n - B_{\mathrm{e}n})$$

$$T_{\mathrm{ST}n} = \frac{1}{3}(2\ \mathrm{Im}\ A_n - B_{\mathrm{e}n})$$

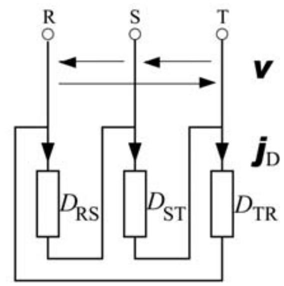
$$T_{\mathrm{TR}n} = -\frac{1}{3}(s\ \sqrt{3}\,\mathrm{Re}\ A_n + \mathrm{Im}\ A_n + B_{\mathrm{e}n})$$

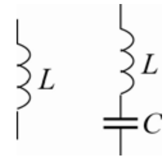
<u>Minimalizacja prądu biernego i prądu niezrównoważenia</u>

Kompensator idealny




Kompensator zredukowany



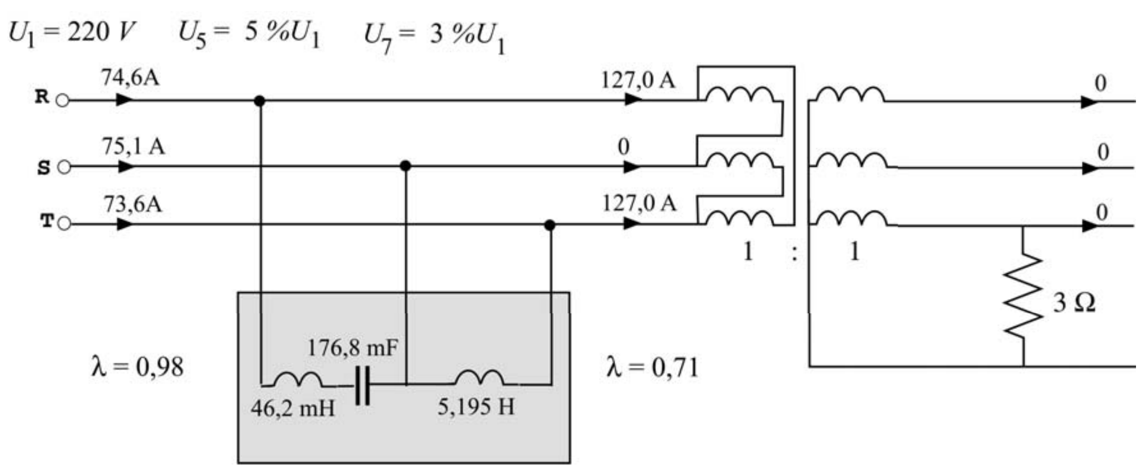

L.S. Czarnecki,

"Minimization of unbalanced and reactive currents in three-phase asymmetrical circuits with nonsinusoidal voltage,"

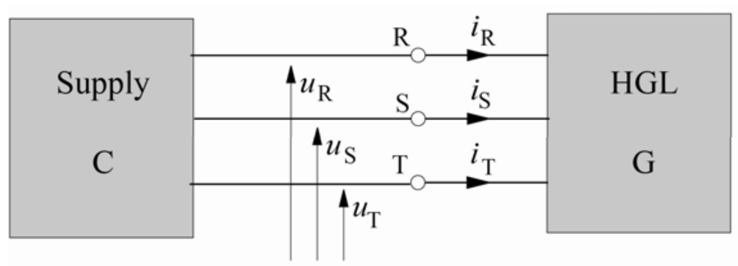
*Proc. IEE,* Vol. 139, Pt. B, No. 4, pp. 347-354, July 1992.

$$d \overset{\mathrm{df}}{=} \| \boldsymbol{j}_{\mathrm{T}} - \boldsymbol{j}_{\mathrm{D}} \|$$

<u>Przykład:</u>



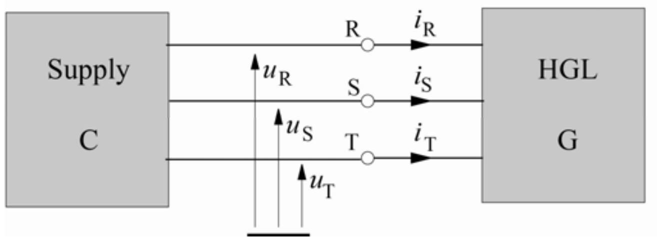
# Składowe fizyczne prądu w obwodach trójfazowych z odbiornikami generującymi harmoniczne

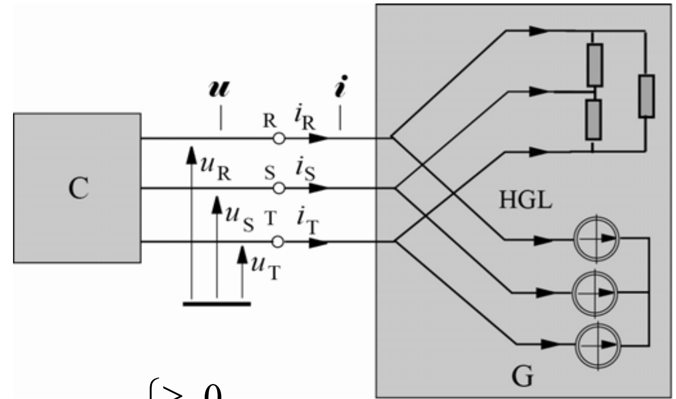



$$P_n = U_{Rn} I_{Rn} \cos\varphi_{Rn} + U_{Sn} I_{Sn} \cos\varphi_{Sn} + U_{Tn} I_{Tn} \cos\varphi_{Tn} \quad \begin{cases} \geq 0 \\ < 0 \end{cases}$$

Jeśli napięcie jesat symetryczne:

$$I_{an} \stackrel{\mathrm{df}}{=} I_{Rn} \cos\varphi_{Rn} + I_{Sn} \cos\varphi_{Sn} + I_{Tn} \cos\varphi_{Tn} \quad \begin{cases} \geq 0 \\ < 0 \end{cases}$$




$$I_{\mathrm{a}n} \stackrel{\mathrm{df}}{=} I_{\mathrm{R}n}\cos\varphi_{\mathrm{R}n} + I_{\mathrm{S}n}\cos\varphi_{\mathrm{S}n} + I_{\mathrm{T}n}\cos\varphi_{\mathrm{T}n} \begin{cases} \geq 0 \\ < 0 \end{cases}$$

Ze wzgledu na kierunek przepływu energii harmonicznych zbiór rzędów harmonicznych $N$ może być rozłożony na dwa podzbiory $N_{\mathrm{C}}$, i $N_{\mathrm{G}}$,

Jesli $I_{\mathrm{a}n} \geq 0, \quad n \in N_{\mathrm{C}}$ $$\sum_{n\in N_{\mathrm{C}}} \boldsymbol{i}_n \stackrel{\mathrm{df}}{=} \boldsymbol{i}_{\mathrm{C}}, \qquad \sum_{n\in N_{\mathrm{C}}} \boldsymbol{u}_n \stackrel{\mathrm{df}}{=} \boldsymbol{u}_{\mathrm{C}}, \qquad \sum_{n\in N_{\mathrm{C}}} P_n \stackrel{\mathrm{df}}{=} P_{\mathrm{C}}$$

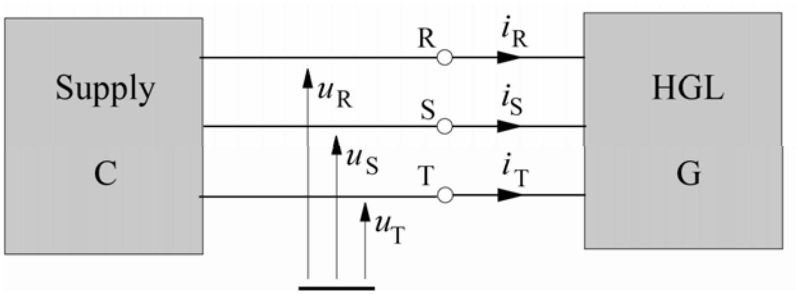
Jesli $I_{\mathrm{a}n} < 0, \quad n \in N_{\mathrm{G}}$ $$\sum_{n\in N_{\mathrm{G}}} \boldsymbol{i}_n \stackrel{\mathrm{df}}{=} \boldsymbol{i}_{\mathrm{G}}, \qquad \sum_{n\in N_{\mathrm{G}}} \boldsymbol{u}_n \stackrel{\mathrm{df}}{=} -\boldsymbol{u}_{\mathrm{G}}, \qquad \sum_{n\in N_{\mathrm{G}}} P_n \stackrel{\mathrm{df}}{=} -P_{\mathrm{G}}.$$

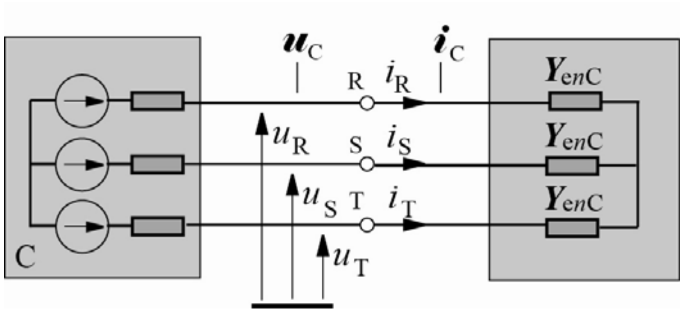
$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{C}} + \boldsymbol{i}_{\mathrm{G}}, \qquad \boldsymbol{u} = \sum_{n\in N} \boldsymbol{u}_n = \boldsymbol{u}_{\mathrm{C}} - \boldsymbol{u}_{\mathrm{G}}, \qquad P = \sum_{n\in N} P_n = P_{\mathrm{C}} - P_{\mathrm{G}}.$$

$$\|\boldsymbol{u}\|^2 = \|\boldsymbol{u}_{\mathrm{C}}\|^2 + \|\boldsymbol{u}_{\mathrm{G}}\|^2$$

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\mathrm{C}}\|^2 + \|\boldsymbol{i}_{\mathrm{G}}\|^2$$

Dla $\quad n \in N_{\mathrm{C}}$



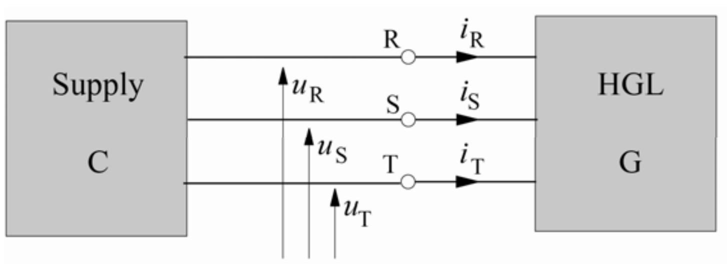




$$\boldsymbol{Y}_{\mathrm{e}n\mathrm{C}} \overset{\mathrm{df}}{=} G_{\mathrm{e}n\mathrm{C}} + jB_{\mathrm{e}n\mathrm{C}} = \frac{\boldsymbol{S}_n^*}{\|\boldsymbol{u}_n\|^2}$$

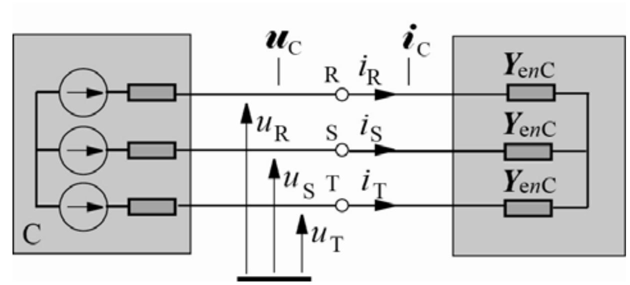
$$\boldsymbol{S}_n \overset{\mathrm{df}}{=} P_n + jQ_n = \boldsymbol{U}_n^T \boldsymbol{I}_n^*$$

$$G_{\mathrm{eC}} \overset{\mathrm{df}}{=} \frac{P_{\mathrm{C}}}{\|\boldsymbol{u}_{\mathrm{C}}\|^2} \qquad \boldsymbol{i}_{\mathrm{aC}} \overset{\mathrm{df}}{=} G_{\mathrm{eC}}\, \boldsymbol{u}_{\mathrm{C}}$$

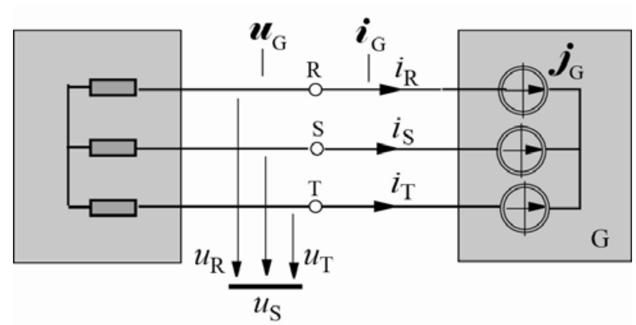
$$\boldsymbol{i}_{\mathrm{C}} = \boldsymbol{i}_{\mathrm{aC}} + \boldsymbol{i}_{\mathrm{sC}} + \boldsymbol{i}_{\mathrm{rC}} + \boldsymbol{i}_{\mathrm{uC}}$$




Dla $n \in N_C$




Dla $n \in N_G$



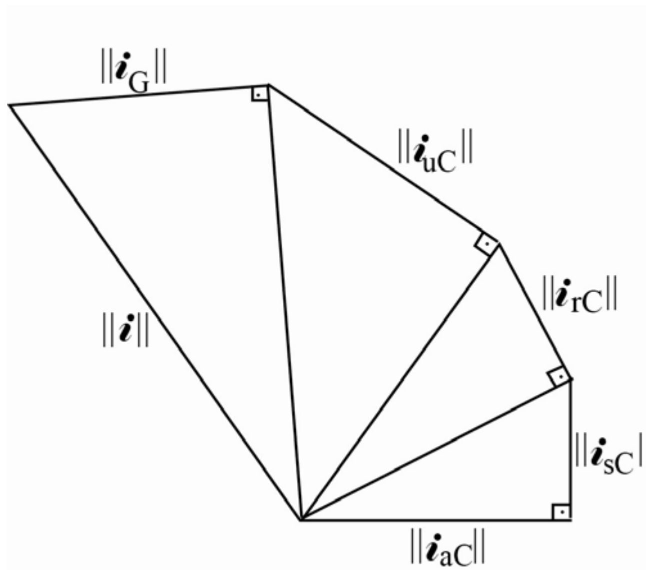

Składowe fizyczne prądu odbiornika generującego harmoniczne:

$$\boldsymbol{i} = \boldsymbol{i}_{aC} + \boldsymbol{i}_{sC} + \boldsymbol{i}_{rC} + \boldsymbol{i}_{uC} + \boldsymbol{i}_{G}$$

Składowe fizyczne są wzajemnie ortogonalne, zatem

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\mathrm{aC}}\|^2 + \|\boldsymbol{i}_{\mathrm{sC}}\|^2 + \|\boldsymbol{i}_{\mathrm{rC}}\|^2 + \|\boldsymbol{i}_{\mathrm{uC}}\|^2 + \|\boldsymbol{i}_{\mathrm{G}}\|^2$$

Ilustracja geometryczna:



,

# Równanie mocy HGL:

$$S \overset{\mathrm{df}}{=} \|\boldsymbol{u}\|\cdot\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{u}_{\mathrm{C}}\|^2 + \|\boldsymbol{u}_{\mathrm{G}}\|^2}\sqrt{\|\boldsymbol{i}_{\mathrm{C}}\|^2 + \|\boldsymbol{i}_{\mathrm{G}}\|^2} = \sqrt{S_{\mathrm{C}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2}$$

$$S_{\mathrm{C}} \overset{\mathrm{df}}{=} \|\boldsymbol{u}_{\mathrm{C}}\|\cdot\|\boldsymbol{i}_{\mathrm{C}}\| = \sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{sC}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{uC}}^2}$$

$$S_{\mathrm{G}} \overset{\mathrm{df}}{=} \|\boldsymbol{u}_{\mathrm{G}}\|\cdot\|\boldsymbol{i}_{\mathrm{G}}\|$$

$$S_{\mathrm{E}} \overset{\mathrm{df}}{=} \sqrt{\|\boldsymbol{u}_{\mathrm{C}}\|^2\|\boldsymbol{i}_{\mathrm{G}}\|^2 + \|\boldsymbol{u}_{\mathrm{G}}\|^2\|\boldsymbol{i}_{\mathrm{C}}\|^2}$$

# Współczynnik mocy:

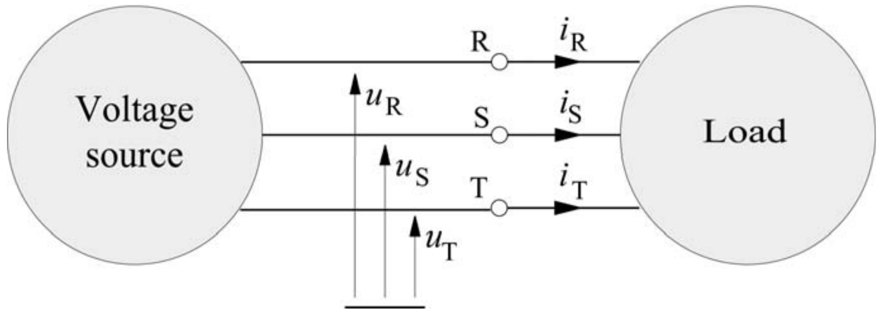
$$\lambda \overset{\mathrm{df}}{=} \frac{P}{S} = \frac{P_{\mathrm{C}} - P_{\mathrm{G}}}{\sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{sC}}^2 + Q_{\mathrm{rC}}^2 + D_{\mathrm{uC}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2}}$$

<u>Uwagi dotyczące:</u>

# Instantaneous Reactive Power p-q Theory

***Akagi, Nabae, Kanazawa***

**(1983)**




Clarke Transform:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix} = C \begin{bmatrix} u_R \\ u_S \end{bmatrix} \qquad \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix} = C \begin{bmatrix} i_R \\ i_S \end{bmatrix}$$

Instantaneous real or active power:

$$p = u_\alpha i_\alpha + u_\beta i_\beta,$$

Instantaneous imaginary or reactive power

$$q = u_\alpha i_\beta - u_\beta i_\alpha$$

## Instantaneous active current:

In $\alpha, \beta$ coordinates:

$$i_{\alpha\mathrm{p}} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} p \qquad i_{\beta\mathrm{p}} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2} p$$

In phase coordinates:

$$\begin{bmatrix} i_{\mathrm{Rp}} \\ i_{\mathrm{Sp}} \end{bmatrix} = C^{-1} \begin{bmatrix} i_{\alpha\mathrm{p}} \\ i_{\beta\mathrm{p}} \end{bmatrix} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_{\alpha\mathrm{p}} \\ i_{\beta\mathrm{p}} \end{bmatrix}$$

## Instantaneous reactive current:

In $\alpha, \beta$ coordinates:

$$i_{\alpha\mathrm{q}} = \frac{-u_\beta}{u_\alpha^2 + u_\beta^2} q \qquad i_{\beta\mathrm{q}} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} q$$

In phase coordinates:

$$\begin{bmatrix} i_{\mathrm{Rq}} \\ i_{\mathrm{Sq}} \end{bmatrix} = C^{-1} \begin{bmatrix} i_{\alpha\mathrm{q}} \\ i_{\beta\mathrm{q}} \end{bmatrix}$$

Według  Nabae'a, który jest głównym autorem
IRP p-q Theory:

*"It was developed to enable instantaneous compensation of the reactive power"*

Rzeczywiście,

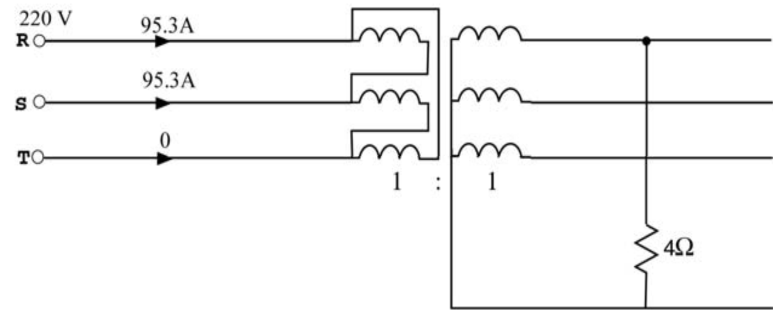
*moce chwilowe p* & *q*
mogą być obliczone momentalnie
co sugeruje wniosek,
że właściwości energetyczne odbiorników mogą być też identyfikowane momentalnie
(z opóźnieniem potrzebnym jedynie do obliczeń)

## *Przykład 1*

$$u_R = \sqrt{2}\,U\cos\omega t, \quad U = 220 \text{ V}$$

$$i_R = \sqrt{2}\,I\cos(\omega t + 30^0), \quad I = 95.3 \text{ A}$$

Q = 0




$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = C\begin{bmatrix} u_R \\ u_S \end{bmatrix} = \begin{bmatrix} \sqrt{3}\,U\cos\omega t \\ \sqrt{3}\,U\sin\omega t \end{bmatrix}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C\begin{bmatrix} i_R \\ -i_R \end{bmatrix} = \begin{bmatrix} \sqrt{3}\,I\cos(\omega t + 30^o) \\ -I\cos(\omega t + 30^o) \end{bmatrix}$$

Instantaneous powers,

Active: $$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}\,U\,I\,[1 + \cos 2(\omega t + 30^0)]$$

Reactive: $$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}\,U\,I\sin 2(\omega t + 30^0)$$

For $2(\omega t + 30^0) = 90^0, \quad p = -q$

Instantaneous currents,

Active: $$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \sqrt{\frac{2}{3}}\,I\,[1 + \cos 2(\omega t + 30^0)]\begin{bmatrix} \cos\omega t \\ \cos(\omega t - 120^0) \end{bmatrix}$$

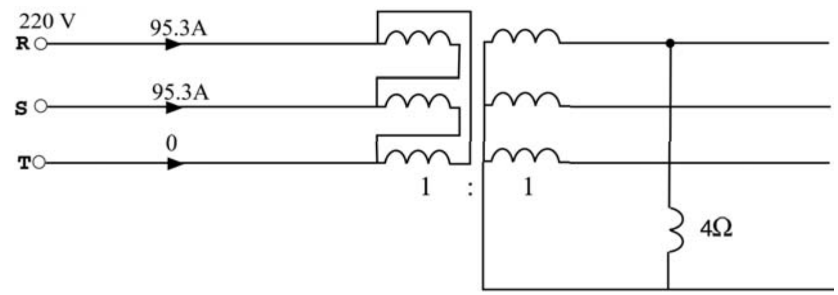
Reactive: $$\begin{bmatrix} i_{Rq} \\ i_{Sq} \end{bmatrix} = \sqrt{\frac{2}{3}}\,I\sin 2(\omega t + 30^0)\begin{bmatrix} \sin\omega t \\ \sin(\omega t - 120^0) \end{bmatrix}$$

**Odbiornik czysto rezystancyjny obciąża źródło chwilowym prądem biernym**

## *Przykład 2*

$$u_R = \sqrt{2}\, U \cos\omega t, \quad U = 220\ \text{V}$$
$$i_R = \sqrt{2}\, I \cos(\omega t - 60^0), \quad I = 95.3\ \text{A}$$

$P = 0$




$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C \begin{bmatrix} i_R \\ -i_R \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, I \cos(\omega t - 60^o) \\ -I \cos(\omega t - 60^o) \end{bmatrix}$$

Instantaneous powers,

Active: $$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}\, U\, I\, [\cos(2\omega t - 30^0)]$$

Reactive: $$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}\, U\, I\, [1 + \sin(2\omega t - 30^0)]$$

For $2\omega t - 30^0 = 0, \quad p = -q$

Instantaneous active current in R line:

$$i_{Rp} = \frac{I}{\sqrt{6}} [\cos(\omega t - 30^0) + \cos(3\omega t - 30^0)]$$

**Odbiornik czysto reaktancyjny obciąża źródło chwilowym prądem czynnym**

Wniosek:

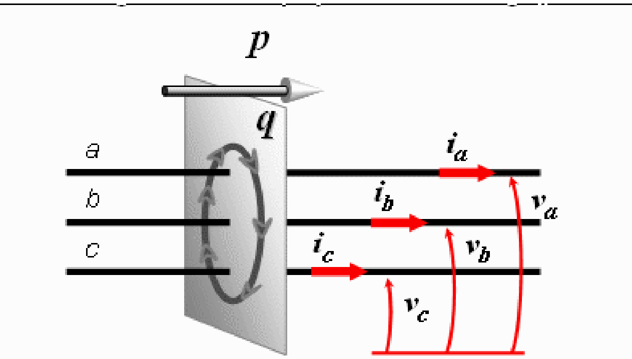
Para mocy chwilowych *p* i *q*, zmierzonych w pewnej chwil t

nie określa właściwości energetycznych odbiornika

H. Akagi, E. H. Watanabe and M. Aredes book
**Instantaneous Power Theory and Applications to Power Conditioning**

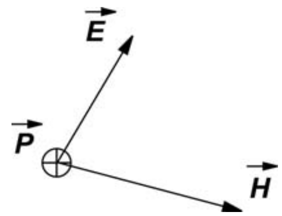
Przedstawili następującą interpretację fizyczną
chwilowej mocy biernej q:

*"…the imaginary power q is proportional to the quantity of energy that is being exchanged between the phases of the system…" "Fig. ( ) summarizes the above explanations about the real and imaginary powers."*

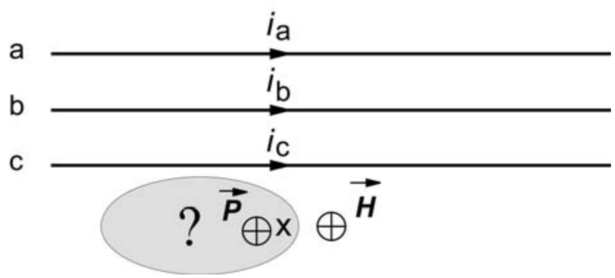



*p : instantaneous total energy flow per time unit;*
*q : energy exchanged between the phases without transferring energy.*
¶

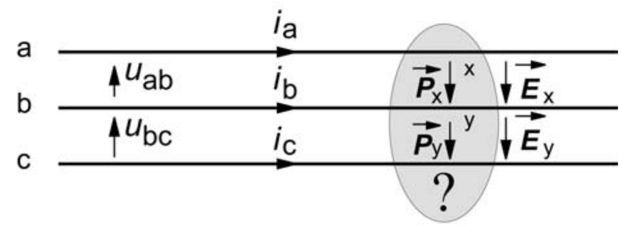
$$\vec{P} = \vec{E} \times \vec{H}$$




$$\iint_{A} \vec{P} \bullet d\vec{A} = \frac{dW(t)}{dt}$$




Wektor Poyinting’a ***P***
nie może być równoległy
do wektora natężenia pola magnetycznego ***H***




Wektor Poyinting’a ***P***
nie może być równoległy
do wektora natężenia pola elektrycznego ***E***

Interpretacja Akagi’ego jest błędna.

Energia nie może wirować wokół linii transmisyjnej bądź przepływać
między przewodnikami liniami

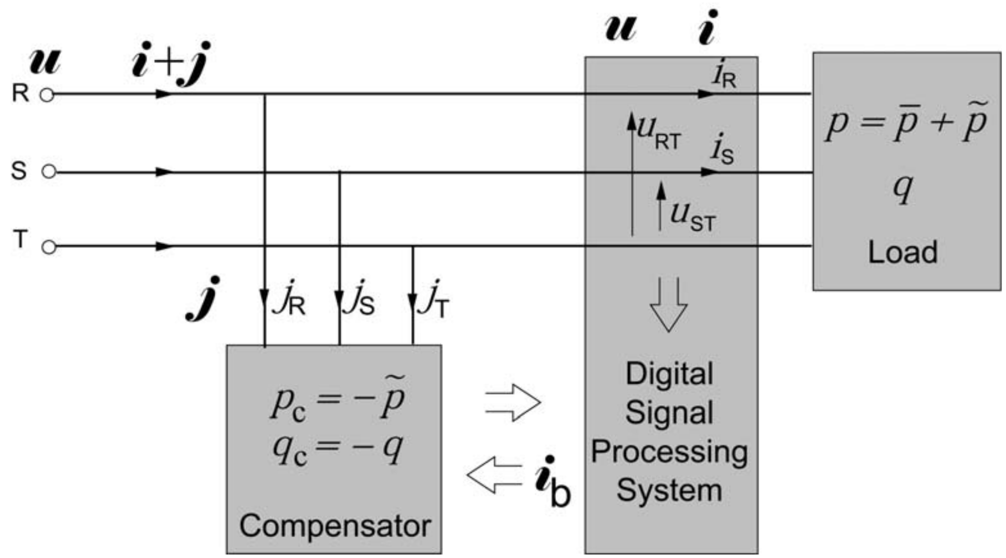
Teoria Chwilowej Mocy Biernej p-q nie ma znaczenia poznawczego
gdyż
nie dostarcza interpretacji zjawisk fizycznych

Dopiero teoria CPC wyjaśniła sens fizyczny mocy $q$
Jest to wielkość złożona

$$q = u_{\alpha} i_{\beta} - u_{\beta} i_{\alpha} = - Q - D \sin(2\omega t + \psi)$$

Ale nawet ten wynik jest poprawny tylko wtedy,
gdy napięcia i prądy są sinusoidalne

## Głównym zastosowaniem Teorii Chwilowej Mocy Biernej (TCPB) p-q, są algorytmy sterowania kompensatorów.

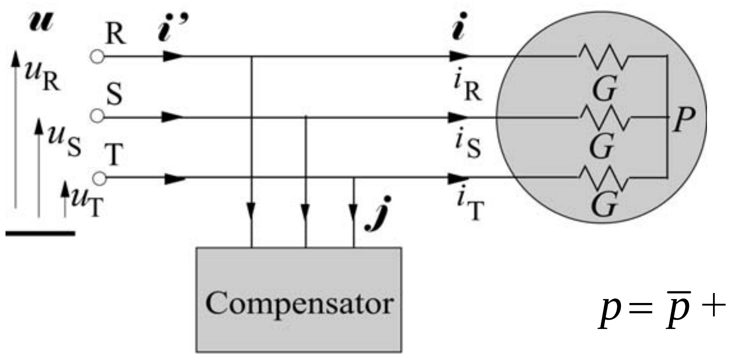



Według TCMB p-q, kompensator ma kompensować chwilową moc bierną *q* i składową oscylacyjną chwilowej mocy czynnej *p*.

**L.S. Czarnecki, (2009) "Effect of supply voltage harmonics on IRP p-q-based switching compensator control"**
***IEEE Trans. on Power Electronics*, Vol. 24, No. 2**

## If the load is an ideal resistive load supplied with <u>nonsinusoidal</u> voltage

$$\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_5$$



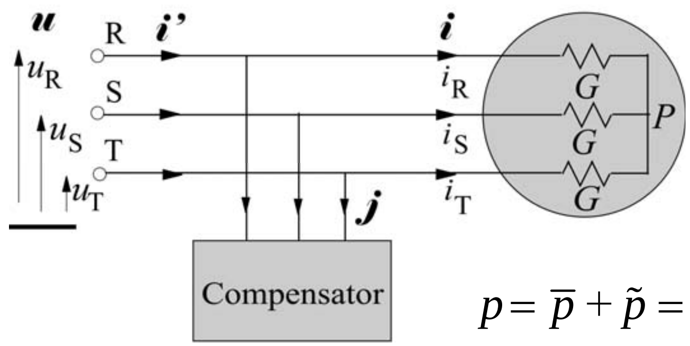

$$p = \bar{p} + \tilde{p} = P_1 + P_5 + 6GU_1U_5\cos 6\omega_1 t$$

$$j = \begin{bmatrix} j_R \\ j_S \end{bmatrix} = \frac{-2\sqrt{2}\,GU_1U_5\cos 6\omega_1 t}{U_1^2 + U_5^2 + 2U_1U_5\cos 6\omega_1 t}\begin{bmatrix} U_1\cos\omega_1 t + U_5\cos 5\omega_1 t \\ U_1\cos(\omega_1 t - 120^0) + U_5\cos(5\omega_1 t + 120^0) \end{bmatrix}$$

**L.S. Czarnecki, "Effect of supply voltage asymmetry on IRP p-q - based switching compensator control"**
***IET Proc. on Power Electronics*, 2010, Vol. 3, No. 1**

**If the load is an ideal resistive load supplied with <u>asymmetrical</u> voltage**

$$\boldsymbol{u} = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}}$$

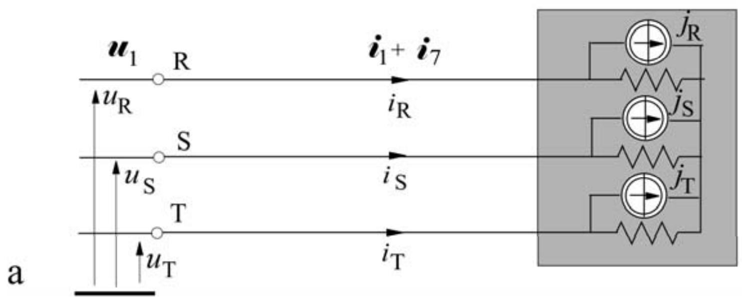


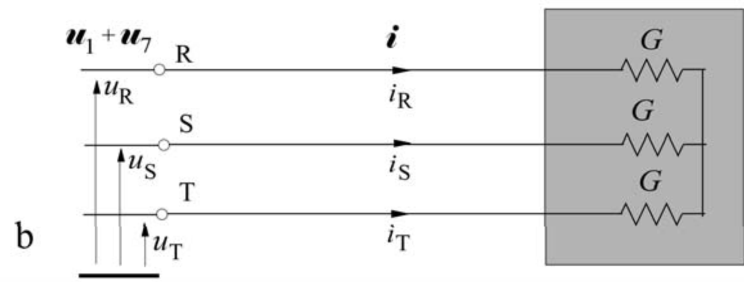

$$p = \overline{p} + \tilde{p} = P^{\mathrm{p}} + P^{\mathrm{n}} + 6GU^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t$$

$$j_{\mathrm{R}} = \sqrt{\frac{2}{3}}\, j_{\alpha} = \frac{-2\sqrt{2}\,G\,(U^{\mathrm{p}} + U^{\mathrm{n}})U^{\mathrm{p}}U^{\mathrm{n}}\cos\omega_1 t\cos 2\omega_1 t}{U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2U^{\mathrm{p}}U^{\mathrm{n}}\cos 2\omega_1 t}$$

$$u_R = \sqrt{2}\,U_1 \cos \omega_1 t,$$
$$i_R = \sqrt{2}\,I_1 \cos \omega_1 t + \sqrt{2}\,I_7 \cos 7\omega_1 t$$

$$u_R = \sqrt{2}\,U_1 \cos \omega_1 t + \sqrt{2}\,U_7 \cos 7\omega_1 t$$
$$\boldsymbol{i} = G\,\boldsymbol{u}$$




$$p = \overline{p} + \tilde{p} = 3U_1 I_1 + 3U_1 I_7 \cos 6\omega_1 t$$
$$q = 0$$

$$p = \overline{p} + \tilde{p} = P + 6GU_1 U_7 \cos 6\omega_1$$
$$q = 0$$

These two circuits,
different
with respect to properties and needed compensation,
are identical
in terms of IRP p-q Theory

Instantaneous powers *p* and *q*
are algebraic forms (AF)
of the supply voltages and the load currents products

$$p = \bar{p} + \tilde{p} = \mathrm{AF}\{u_{\mathrm{r}}\, i_{\mathrm{s}}\}$$

SUPPLY LOAD r , s = R, S or T

$$q = \bar{q} + \tilde{q} = \mathrm{AF}\{u_{\mathrm{r}}\, i_{\mathrm{s}}\}$$

Values of *p* and *q* powers do not provide information
whether their properties come
from the supply voltage or from the load current

# ROBOCZA, ODBITA I SZKODLIWA MOC CZYNNA

Podstawową wielkością w rozliczeniach energetycznych
jest energia dostarczana do jej użytkownika

$$\int_0^\tau P\,dt = W_\mathrm{a}\,,$$

$P -$ moc czynna, $\tau -$ okres rozliczeniowy

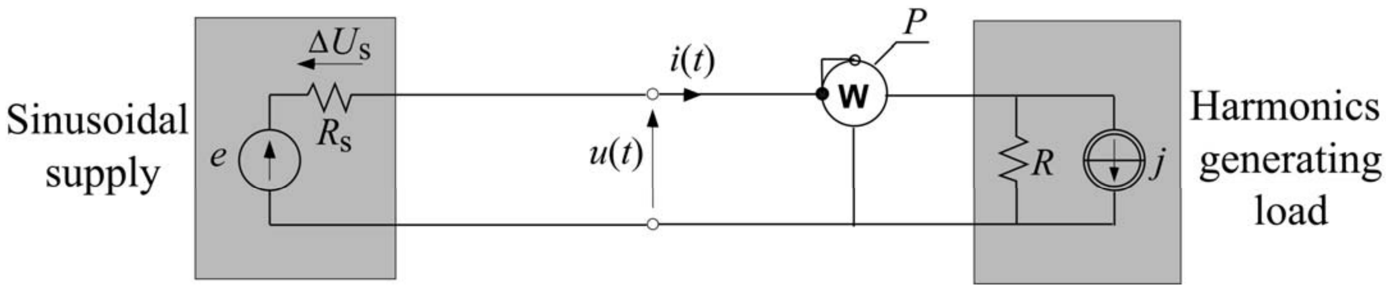
Terminy ***moc czynna***, $P$ oraz ***moc użyteczna***
są zwykle traktowane jako synonimy

Energia czynna $W_\mathrm{a}$ jest traktowana zwykle jako energia użyteczna

Odbiorcy wielkich ilości energii pokrywają też zwykle dodatkowe koszty
wynikające z niskiego współczynika mocy $\lambda$.

Takie podejście do rozliczeń energetycznych
pojawiło się na przełomie
XIX i XX wieku
i obowiązuje do chwili obecnej

Przez większość tego okresu energia elektryczna dostarczana
była z generatorów synchronicznych
produkujących niemal doskonale sinusoidalne i symetryczne
napięcie trójfazowe
i zużywana była w dominującej części
przez odbiorniki liniowe




$$u(t) = \sum_{n \in N} u_n = u_1 + u_h \qquad i(t) = \sum_{n \in N} i_n = i_1 + i_h$$

$$P = \frac{1}{T}\int_0^T u(t)\, i(t)\, dt = P_1 + P_2 + P_3 + P_4 + \ldots$$

$$P_1 = U_1 I_1 > 0$$

$$P_n = U_n I_n = (-R_s I_n) I_n = -R_s I_n^2 < 0$$

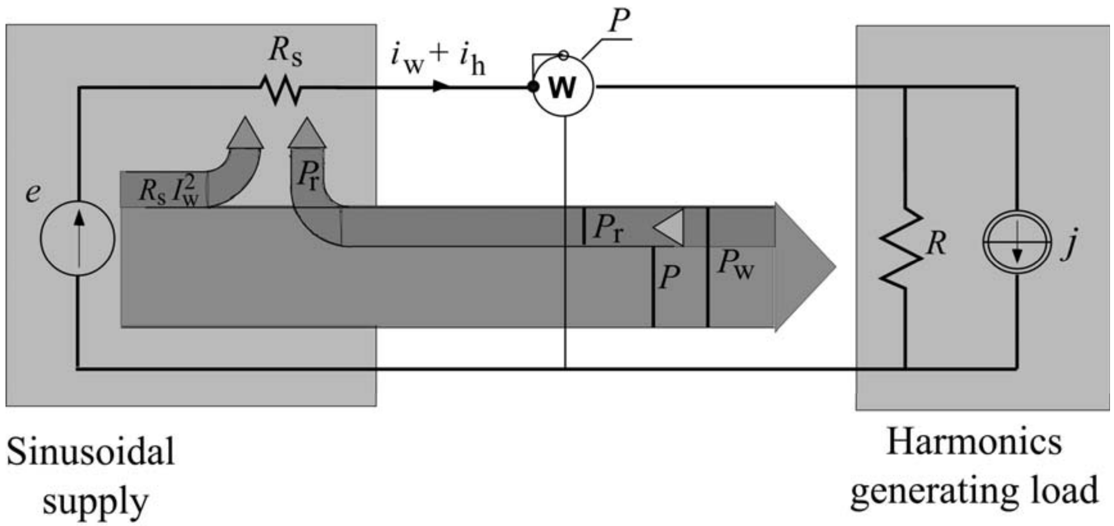
Energia do odbiornika dostarczana jest z mocą $P_1$.
jest to

**robocza moc czynna, $P_1 = P_w$**

$$-(P_2 + P_3 + P_4 + \ldots) = P_r$$

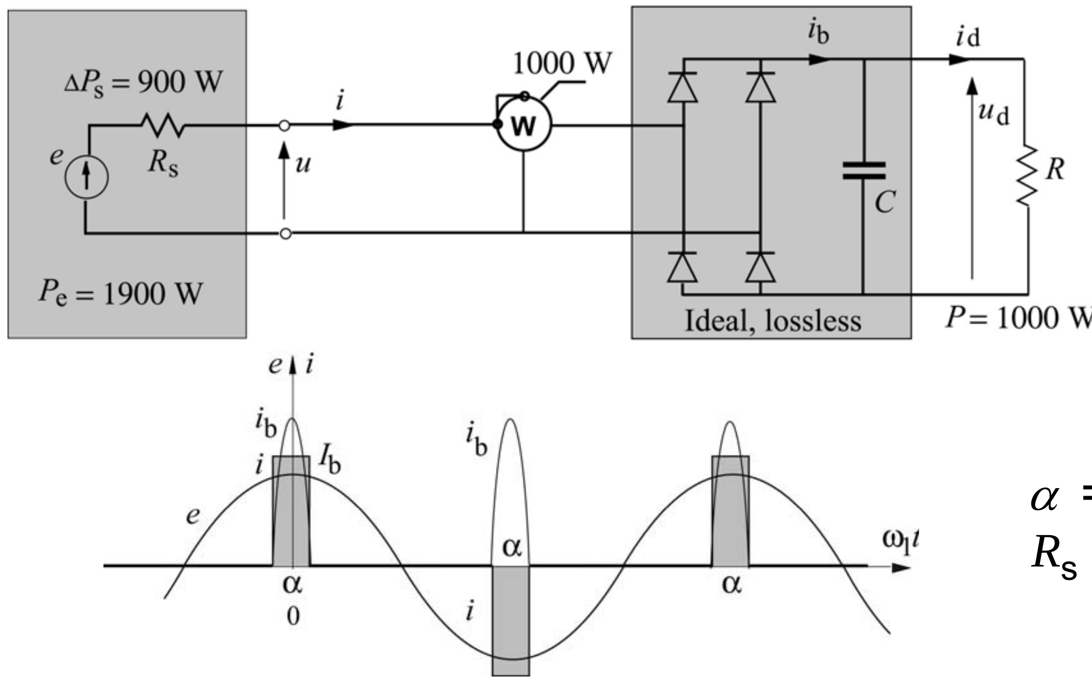
**odbita moc czynna**

$$P = P_w - P_r$$




*Odbiornik o mocy czynnej $P$ generujący harmoniczne musi być zasilany z mocą roboczą $P_w$.*

$$P_w > P$$




$\alpha$ = 10 deg,
$R_s$ = 5% of $R$

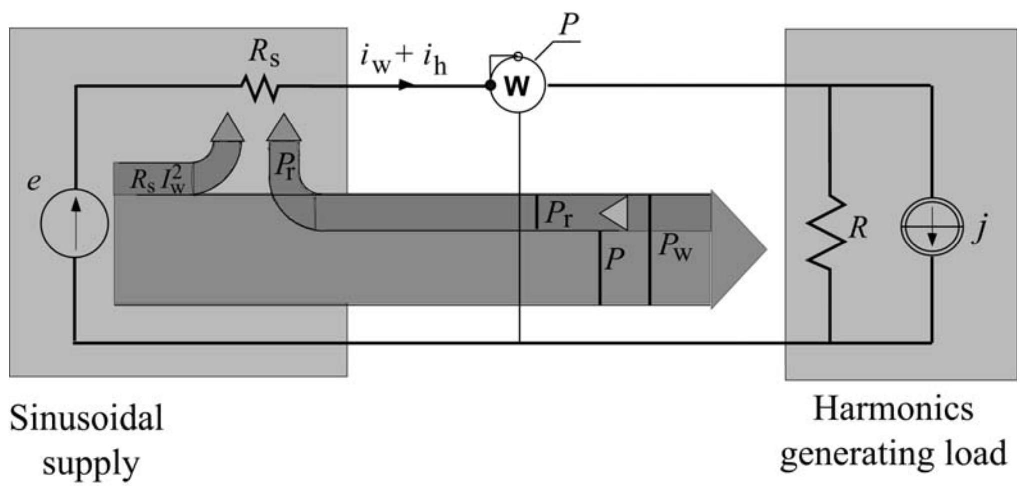
| | |
|---|---|
| Moc czynna | $P = 1000$ W |
| Robocza moc czynna | $P_w = 1536$ W |
| Odbita moc czynna | $P_r = 536$ W |
| Moc strat w zasilaniu: | $\Delta P_s = 900$ W |

Harmoniczne generowane w odbiorniku
powodują dodatkowe straty wewnątrz systemu zasilającego.
Odbiorca winien płacić za energię roboczą

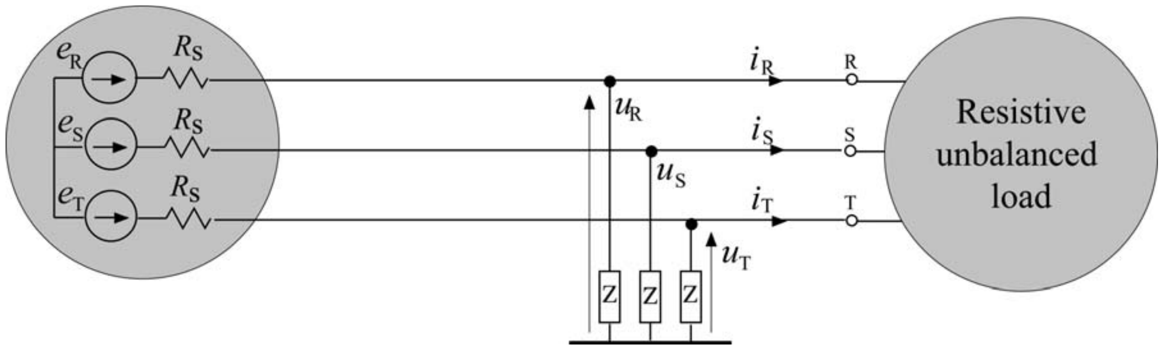
$$\int_0^\tau P_\mathrm{w}\,dt = \int_0^\tau (P + P_\mathrm{r})\,dt = W_\mathrm{w} > W_\mathrm{a}$$

Prąd potrzebny do przenoszenia energii roboczej $W_\mathrm{w}$
ma większą moc skuteczną
od prądu przenoszącego energię czynną $W_\mathrm{a}$

$$\Delta P_\mathrm{s} = P_\mathrm{r} + R_\mathrm{s}\, I_\mathrm{w}^2$$



# Robocza i odbita moc czynna w układach trójfazowych, trójprzewodowych

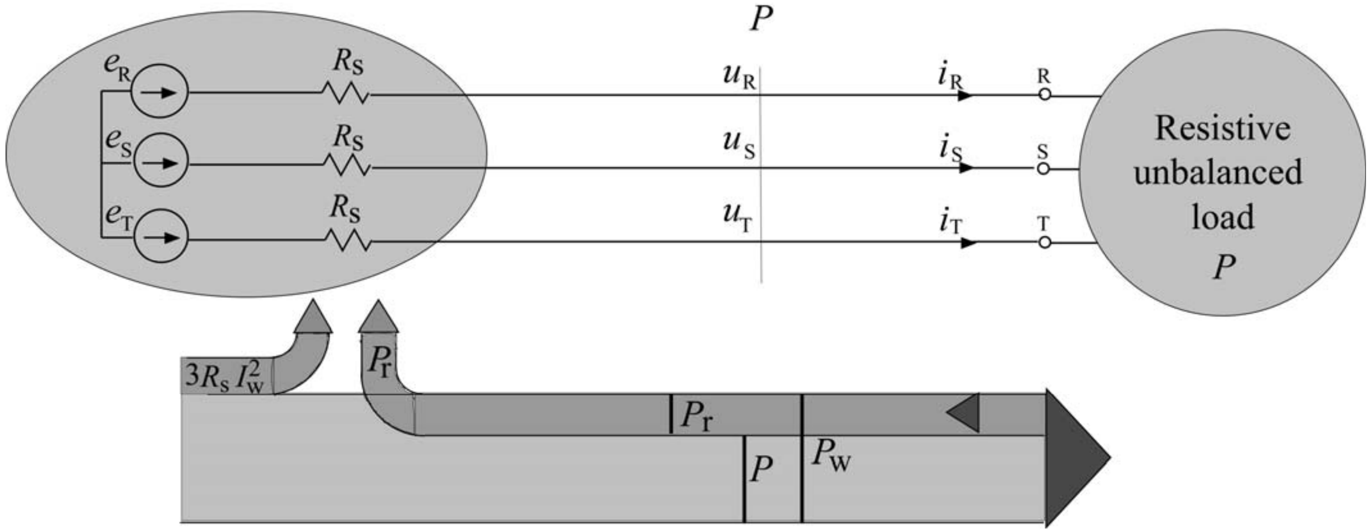



$$\begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} \overset{\text{df}}{=} \boldsymbol{u} = \boldsymbol{u}^{\text{p}} + \boldsymbol{u}^{\text{n}} \qquad \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} \overset{\text{df}}{=} \boldsymbol{i} = \boldsymbol{i}^{\text{p}} + \boldsymbol{i}^{\text{n}}$$

$$P = \frac{1}{T}\int_0^T \boldsymbol{u}^{\text{T}}\boldsymbol{i}\,dt = (\boldsymbol{u},\boldsymbol{i}) = (\boldsymbol{u}^{\text{p}},\boldsymbol{i}^{\text{p}}) + (\boldsymbol{u}^{\text{n}},\boldsymbol{i}^{\text{n}}) = P^{\text{p}} + P^{\text{n}}$$

$$P^{\text{n}} \overset{\text{df}}{=} (\boldsymbol{u}^{\text{n}},\boldsymbol{i}^{\text{n}}) = (-R_{\text{s}}\,\boldsymbol{i}^{\text{n}},\boldsymbol{i}^{\text{n}}) = -R_{\text{s}}\,\|\boldsymbol{i}^{\text{n}}\|^2 = -P_{\text{r}} < 0$$

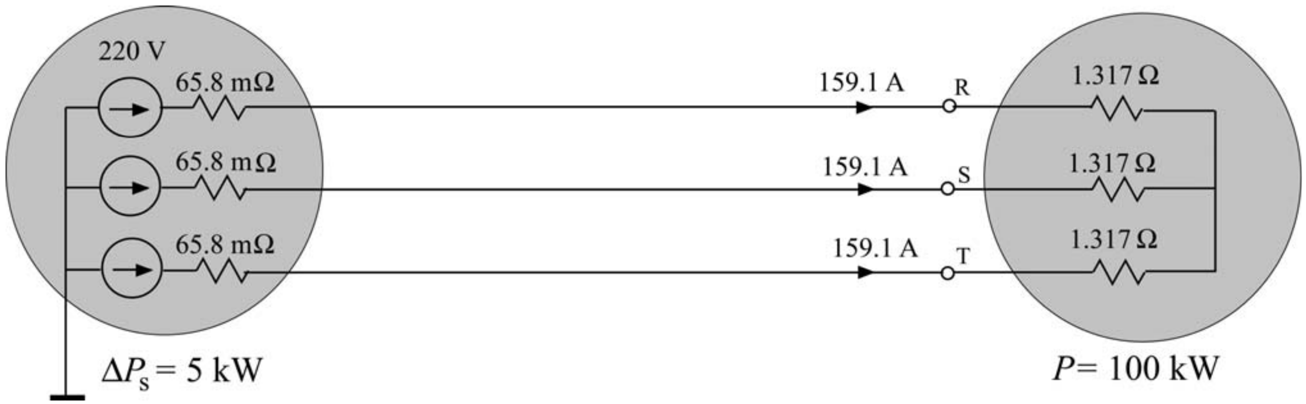
$$P = P^{\text{p}} + P^{\text{n}} \overset{\text{df}}{=} P_{\text{w}} - P_{\text{r}}$$




Odbiornik niezrównoważony o mocy czynnej $P$

musi być zasilany

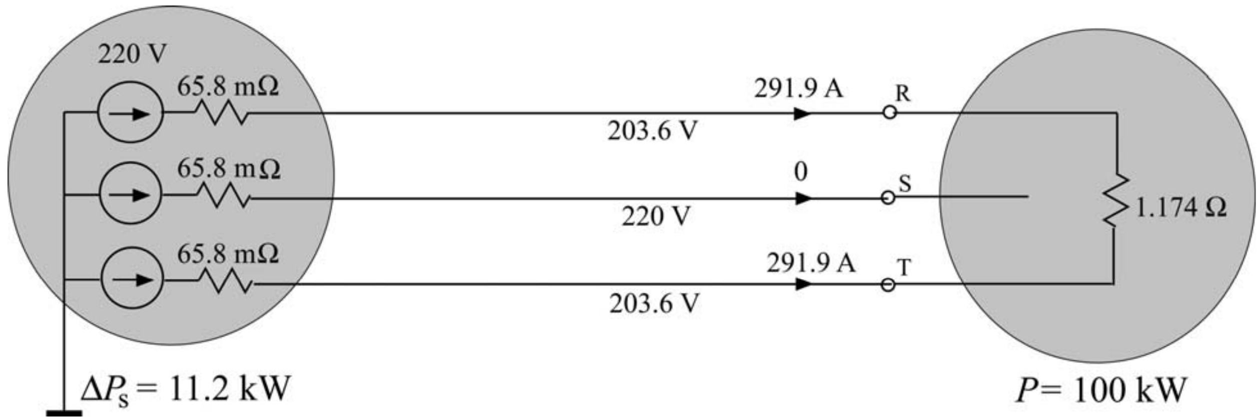
roboczą mocą czynną $P_{\mathrm{w}}$ większą od mocy czynnej




$$\|\boldsymbol{i}\| = \sqrt{3} \times 159.1 = 275.6\text{A}$$

Moc strat w źródle

$$\Delta P_s = R_s \|\boldsymbol{i}\|^2 = 0.0658 \times 275.6^2 = 5.0\,\text{kW}$$




$$\begin{bmatrix} \boldsymbol{U}^{\mathrm{p}} \\ \boldsymbol{U}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1, & \alpha, & \alpha^* \\ 1, & \alpha^*, & \alpha \end{bmatrix}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{S}} \\ \boldsymbol{U}_{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} 208.9e^{j0^0} \\ 11.1e^{j120^0} \end{bmatrix}\mathrm{V}$$

$$\begin{bmatrix} \boldsymbol{I}^{\mathrm{p}} \\ \boldsymbol{I}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1, & \alpha, & \alpha^* \\ 1, & \alpha^*, & \alpha \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_{\mathrm{R}} \\ \boldsymbol{I}_{\mathrm{S}} \\ \boldsymbol{I}_{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} 168.5e^{j0^0} \\ 168.5e^{-j60^0} \end{bmatrix}\mathrm{A}$$

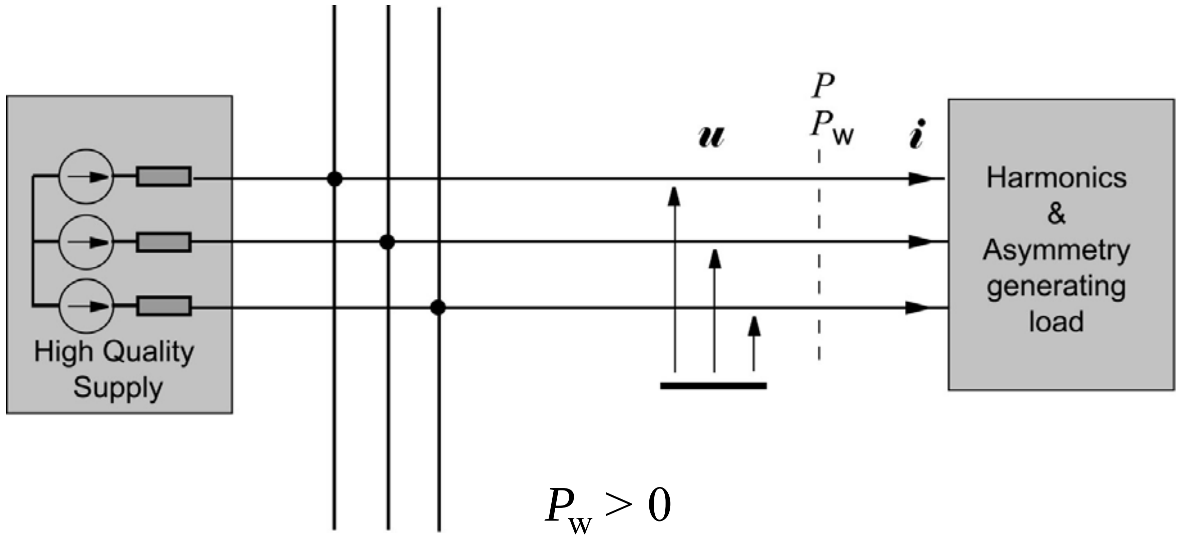
$$P_{\mathrm{w}} = P^{\mathrm{p}} = (\boldsymbol{u}^{\mathrm{p}}, \boldsymbol{i}^{\mathrm{p}}) = 3U^{\mathrm{p}}\, I^{\mathrm{p}} = 3\times 208.9\times 168.5 = 105.6\ \mathrm{kW}$$

$$P_{\mathrm{r}} = -P^{\mathrm{n}} = -(\boldsymbol{u}^{\mathrm{n}}, \boldsymbol{i}^{\mathrm{n}}) = -3\mathrm{Re}\{\boldsymbol{U}^{\mathrm{n}}\boldsymbol{I}^{\mathrm{n}*}\} = 3\times 11.1\times 168.5 = 5.6\ \mathrm{kW}$$

$$||\boldsymbol{i}_{\mathrm{w}}|| = \frac{P_{\mathrm{w}}}{||\boldsymbol{u}_{\mathrm{w}}||} = \frac{105.6\times 10^3}{\sqrt{3}\times 208.9} = 291.8\,\mathrm{A}$$

## Moc strat w źródle

$$\Delta P_{\mathrm{s}} = R_{\mathrm{s}}||\boldsymbol{i}_{\mathrm{w}}||^2 + P_{\mathrm{r}} = 0.0658\times 291.8^2 + 5.6 = 11.2\ \mathrm{kW}$$



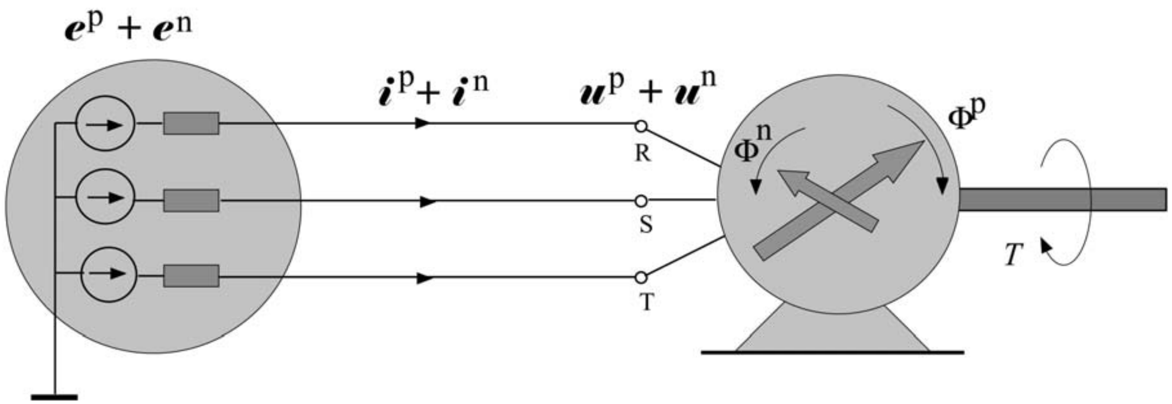

$$P_{\mathrm{w}} > 0$$

$$P_{\mathrm{r}} < 0$$

$$P_{\mathrm{w}} > P$$

Rozliczenia energetyczne, których rdzeniem byłby koszt energii roboczej obciążałyby odbiorcę kosztem strat powodowanych asymetrią prądów i harmonicznymi generowanymi w odbiorniku

Rozważmy maszynę indukcyjną zasilaną
napięciem asymetrycznym




Energia przenoszona przez składową kolejności przeciwnej nie jest przekształcana na energię mechaniczną

Tak samo jest wtedy, gdy napięcie zasilania jest odkształcone.

$$P_w = P^p > 0$$

$$P_r = P^n > 0$$

$$P_w < P$$

Strata DOCHODÓW
dostawcy energii elektrycznej
sprzedawanej odbiorcom
powodującym odkształcenie prądu i jego asymetrię
jest proporcjonalna
do różnicy między mocą roboczą a mocą czynną

$$\Delta P = P_{\mathrm{w}} - P$$

PRZEPŁATA kosztu energii

odbiorcy zasilanego napięciem odkształconym i asymmetrycznym
jest proporcjonalna
do różnicy między mocą czynną a mocą roboczą

$$\Delta P = P - P_{\mathrm{w}}$$

Wtedy, gdy podstawą rozliczeń energetycznych
jest energia czynna, $W_{\mathrm{a}}$,
strona powodująca odkształcenia i asymmetrię
**nie jest finansowo odpowiedzialna**
**za ich skutki**

Wtedy, gdy podstawą rozliczeń energetycznych
jest energia robocza, $W_{\mathrm{w}}$,
strona powodująca odkształcenia i asymmetrię
**płaci za ich skutki**

**Rozliczenia energetyczne oparte na koszcie energii roboczej, $W_{\mathrm{w}}$**
**mogłyby tworzyć motywacje ekonomiczne**
**do poprawy jakości zasilania i do poprawy jakości obciążenia**
**a tym samym,**
**do oszczędności energii**

Pomiar energii roboczej
wymaga analizy harmonicznej,
ograniczonej jednak do harmonicznej podstawowej prądu i napięcia

Systemy energetyczne będą się rozwijały
w kierunku systemów inteligentnych,
**„smart grids”,**
wyposażonych w mieniki
**zdolne do cyfrowej analizy sygnałów, DSP**

**Pomiar energii roboczej $W_{\mathrm{w}}$ przez takie mierniki będzie tylko zmianą**
**na poziomie programowym**

**Główną przeszkodą**
**dla racjonalizacji podstaw rozliczeń energetycznych**
**mogą być**
**- stuletnia tradycja tych rozliczeń na podstawie energii czynnej,**
**- system przepisów, norm i standardów**
**- inercja intelektualna**

**Nie oznacza to jednak,**
**że nie warto podejmować działań**
**w tym kierunku.**
**Systemy energetyczne będą w najbliższej przyszłości**
**podlegały głębokim zmianom.**
**Podstawy rozliczeń energetycznych powinny być jedną z nich**

# Compensation goals
# in systems
# with nonsinusoidal voltages and currents

*Compensators & filters*

are used for

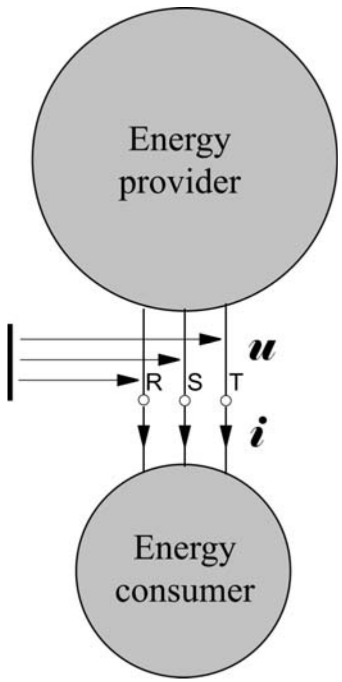
the electrical power system (providers & customers together)

*performance improvement*

&

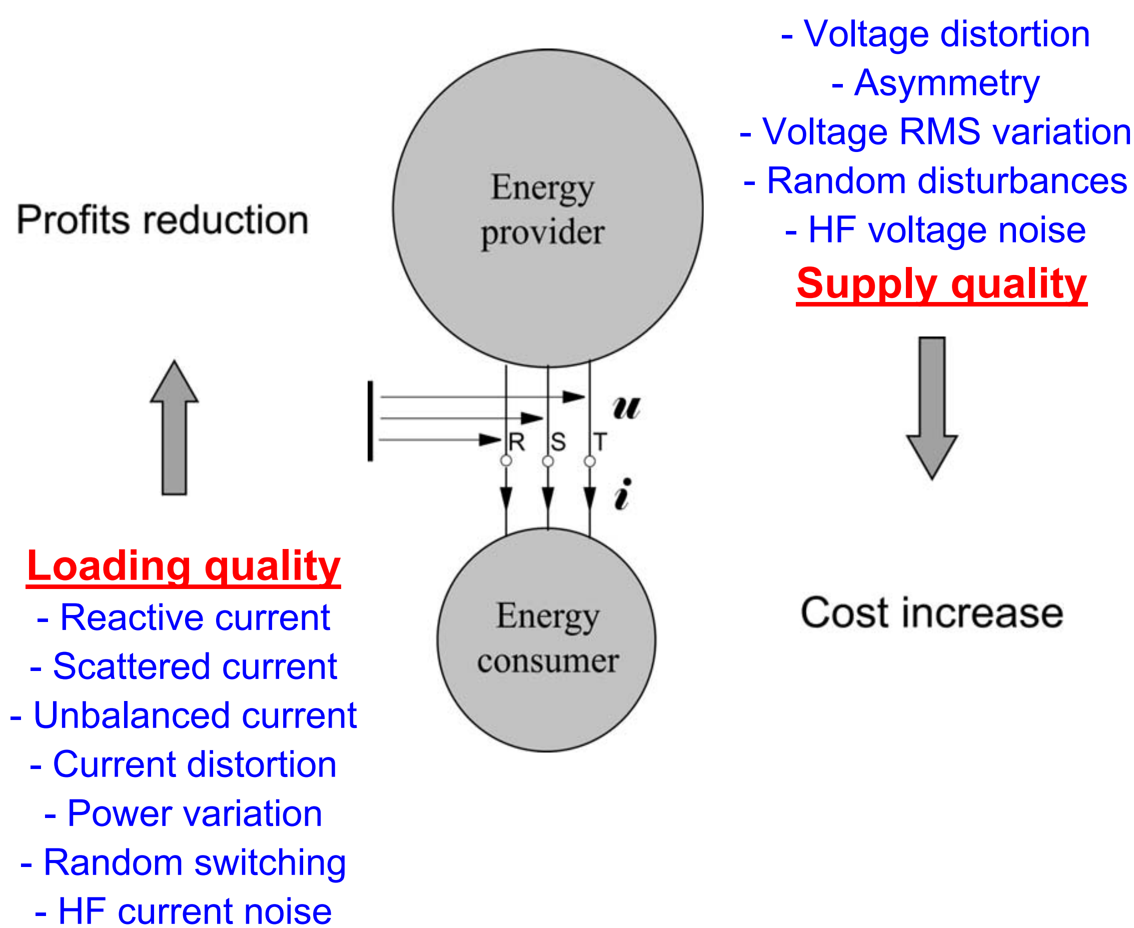
*economic benefits*

There are
two different entities
with different and conflicting goals:



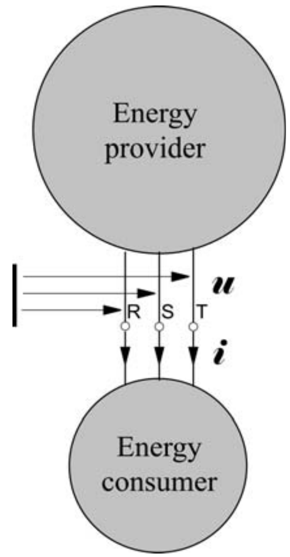

Profits maximization at energy delivery

Cost of energy use minimization



- Voltage distortion
- Asymmetry
- Voltage RMS variation
- Random disturbances
- HF voltage noise
Supply quality
Profits reduction
Energy provider
u
R
S
T
i
Energy consumer
Cost increase
Loading quality
- Reactive current
- Scattered current
- Unbalanced current
- Current distortion
- Power variation
- Random switching
- HF current noise

Profits reduction




- Voltage distortion
- Asymmetry
- Voltage RMS variation
- Random disturbances
- HF voltage noise

**<u>Supply quality</u>**

**<u>Loading quality</u>**

- Reactive current
- Scattered current
- Unbalanced current
- Current distortion
- Power variation
- Random switching
- HF current noise

Cost increase

**Compensation goals in a very essence are economic,**
**unfortunately,**
**we usually are not able to use optimization procedures for compensator control.**

**It is difficult to express *profits reduction* in terms of *loading quality* factors**
**It is difficult to express *cost increase* in terms of *supply quality* factors**

**Compensator cost (investment & operation) is also a component in optimization procedure**

Therefore,

compensation goals
are formulated usually as a reduction of some harmful agents
of the loading quality or/and the supply quality

to a minimum value,
or to a level imposed by standards

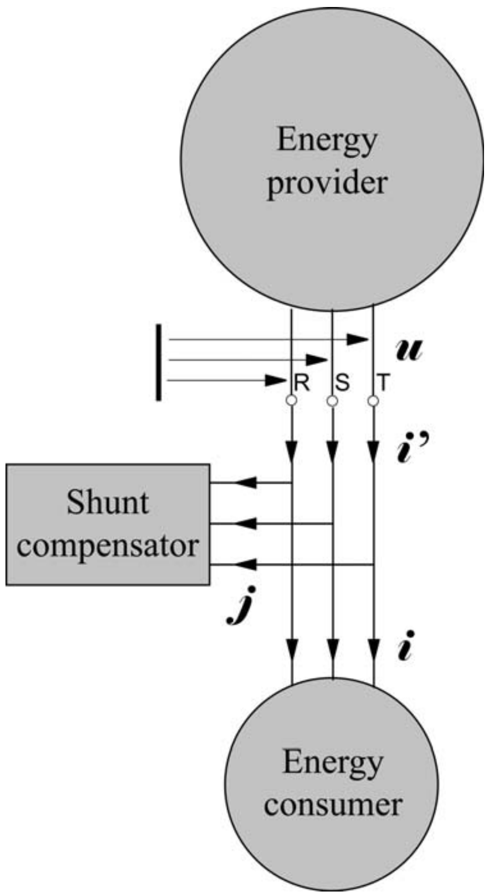
**Usually compensators are used for improvement of degraded loading quality**

Compensators are usually used for improvement of degraded loading quality

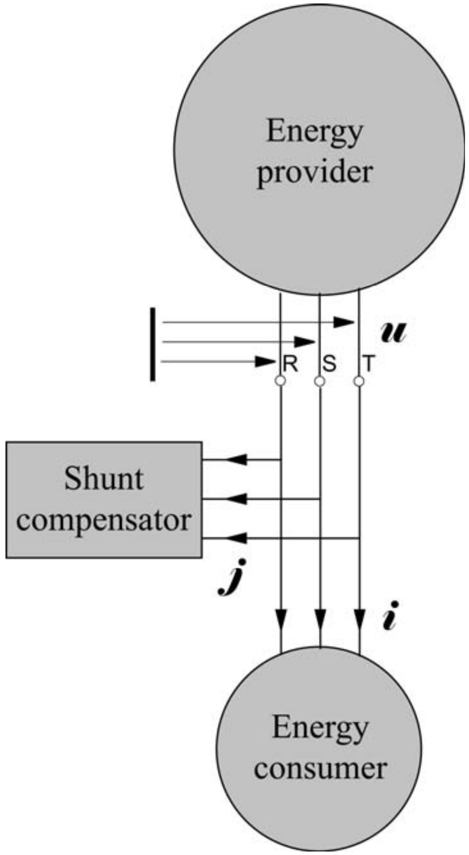
**Loading quality**

- Reactive current
- Scattered current
- Unbalanced current
- Current distortion
- Power variation
- Random switching
- HF current noise

Shunt compensators are needed for that




Reduction
of the supply current three-phase
RMS value
$||\boldsymbol{i}'||$
and its distortion
is a common objective
of shunt compensation




- Voltage distortion
- Asymmetry
- Voltage RMS variation
- Random disturbances
- HF voltage noise

**Degraded supply quality**

Compensator should be able to reach
the specified goals
even at
degraded supply quality

This could be particularly important
***in micro-grids,***
which might be weak systems
with sources of voltage distortion on the provider side
with dominating single-phase
Harmonic Generating Loads

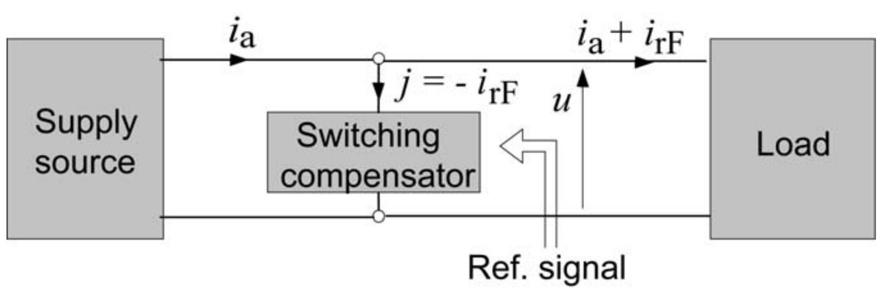
The active current
according to Fryze power theory
is the smallest current of a load which at voltage *u*(*t*)
has the active power *P*

$$i_{\mathrm{a}}(t) = G_{\mathrm{e}}\, u(t), \qquad\qquad G_{\mathrm{e}} = \frac{P}{||u||^2}.$$

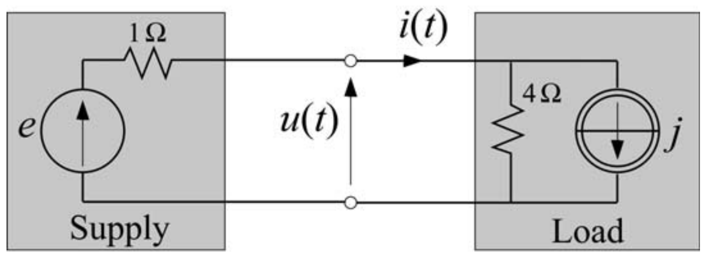
The remaining current,

$$i_{\mathrm{rF}}(t) \overset{\mathrm{df}}{=} i(t) - i_{\mathrm{a}}(t)$$

increases only the supply current RMS value and can be compensated



$$e = 100\sqrt{2}\,\sin\omega_1 t \text{ V}$$




$$j = 50\sqrt{2}\,\sin 3\omega_1 t \text{ A}$$

$$i = \sqrt{2}(20\,\sin\omega_1 t + 40\,\sin 3\omega_1 t)\text{ A}$$

$$u = \sqrt{2}(80\,\sin\omega_1 t - 40\,\sin 3\omega_1 t)\text{ V}$$

$$P = P_1 + P_3 = 1600 - 1600 = 0$$

$$\|i_a\| \equiv 0$$

What is the objective of compensation in this circuit with zero active current

**?**

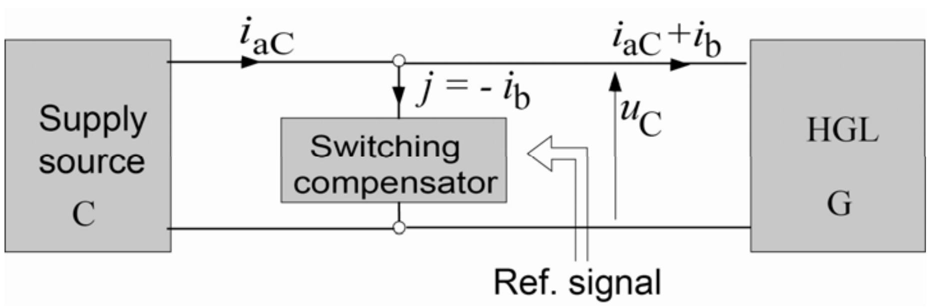
The CPC power theory, unlike Fryze, differentiates two directions of energy flow.
- One, caused by the distribution voltage harmonics, $P_n > 0$
- Second, caused by the load generated current harmonics, $P_n < 0$

The active current
according to CPC power theory
is the smallest current of a load which at distribution system originated voltage $u_D(t)$
has the active power $P_D$

$$i_{aC}(t) \stackrel{df}{=} G_{eC}\, u_C(t) \qquad\qquad G_{eC} \stackrel{df}{=} \frac{P_C}{\|u_C\|^2}$$

The useless and harmful current

$$i_b = i - i_{aC} = i_{rC} + i_{sC} + i_G$$



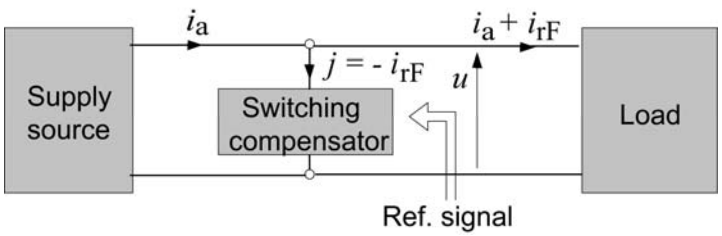
When the system is weak
then shunt compensator affects
the load voltage
and
compensation conditions

Compensation can be achieved only in a recursive process

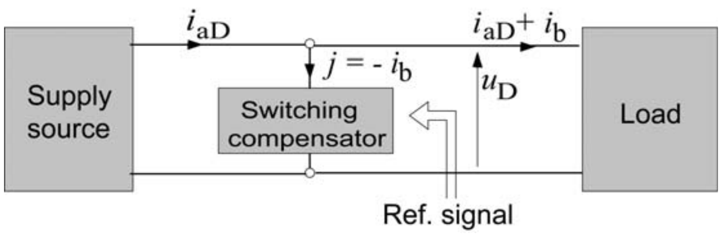
When the system is strong,
shunt compensator does not affect the load voltage

Compensation can be achieved in a single step

Fryze PT approach




CPC PT approach




Recursive process of compensation
converges
to the active current as defined
in the CPC power theory

$$i_{aC}(t) \stackrel{\text{df}}{=} G_{eC}\, u_C(t)$$

$$G_{eC} \stackrel{\text{df}}{=} \frac{P_C}{\|u_C\|^2}$$

The active current, both according to Fryze and CPC PTs,
reproduces
the supply voltage distortion and asymmetry

$$i_{\mathrm{a}}(t) = G_{\mathrm{e}}\, u(t) \qquad\qquad i_{\mathrm{aC}}(t) = G_{\mathrm{eC}}\, u_{\mathrm{C}}(t)$$

$$\boldsymbol{i}_{\mathrm{a}}(t) = G_{\mathrm{e}}\, \boldsymbol{u}(t) \qquad\qquad \boldsymbol{i}_{\mathrm{aC}}(t) = G_{\mathrm{eC}}\, \boldsymbol{u}_{\mathrm{C}}(t)$$

In some situations,
the compensator has to increase
the supply current distortion and asymmetry

There are opinions
that the supply current after compensation
should have
not only the minimum RMS value,
but also be sinusoidal and symmetrical

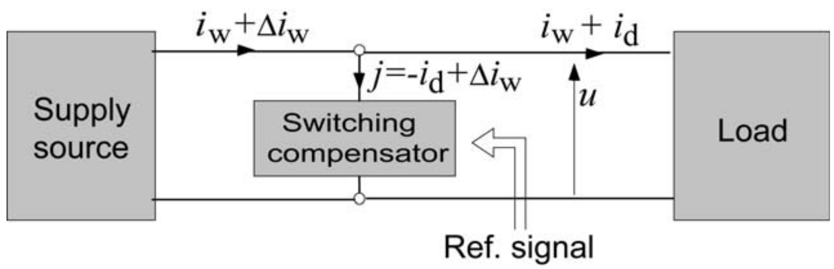
Such a current is referred to as
*working current,*
$i_{\mathrm{w}}$ or $\boldsymbol{i}_{\mathrm{w}}$

Sometimes shunt compensators are controlled
to achieve such a goal

$$i_{\mathrm{w}}(t) \stackrel{\mathrm{df}}{=} i_{\mathrm{a}1}(t) = G_{\mathrm{w}}\, u_1(t) \qquad\qquad G_{\mathrm{w}} \stackrel{\mathrm{df}}{=} G_1 \stackrel{\mathrm{df}}{=} \frac{P_1}{\|u_1\|^2}$$

The remaining part of the current can be referred to as a *detrimental* current

$$i_{\mathrm{d}}(t) \stackrel{\mathrm{df}}{=} i(t) - i_{\mathrm{w}}(t)$$




When a compensator is to reduce the supply current to the active current (according to Fryze or CPC PTs definitions)

then

the compensator current $j$ is orthogonal to the supply voltage $u$.

There is no permanent energy flow to the compensator

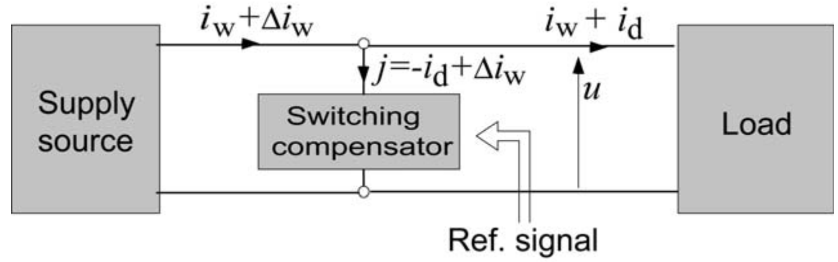
**This is not the case when the working current $i_{\mathrm{w}}$ is the goal of compensation**

The active power of a switching compensator
of purely resistive load is

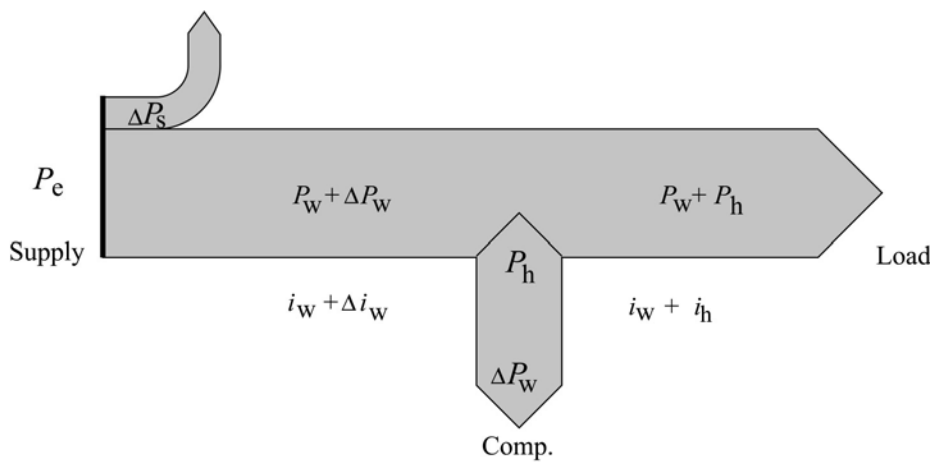
$$P_{\mathrm{c}} = (u, j) = (u, -i_{\mathrm{d}}) = (u, -(i - i_1)) = -(u, i) + (u, i_1) = -P + P_1 = -P_{\mathrm{h}},$$

Compensator has to deliver energy to the system
at the rate of $P_{\mathrm{h}}$

Switching compensators are not active,
but passive devices,
thus
this energy has to be delivered to the compensator
by the current fundamental harmonic



Supply source
$i_w + \Delta i_w$
$j = -i_d + \Delta i_w$
Switching compensator
$i_w + i_d$
$u$
Load
Ref. signal

$\Delta P_s$
$P_e$
Supply
$P_w + \Delta P_w$
$P_w + P_h$
Load
$P_h$
$i_w + \Delta i_w$
$i_w + i_h$
$\Delta P_w$
Comp.

## Three-phase, three-wire systems

$$\boldsymbol{e} = \boldsymbol{e}_1^{\mathrm{p}} + \boldsymbol{e}_1^{\mathrm{n}} + \boldsymbol{e}_{\mathrm{h}}$$

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{w}} + \boldsymbol{i}_{1\mathrm{r}}^{\mathrm{p}} + \boldsymbol{i}_1^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{h}} \stackrel{\mathrm{df}}{=} \boldsymbol{i}_{\mathrm{w}} + \boldsymbol{i}_{\mathrm{d}}$$

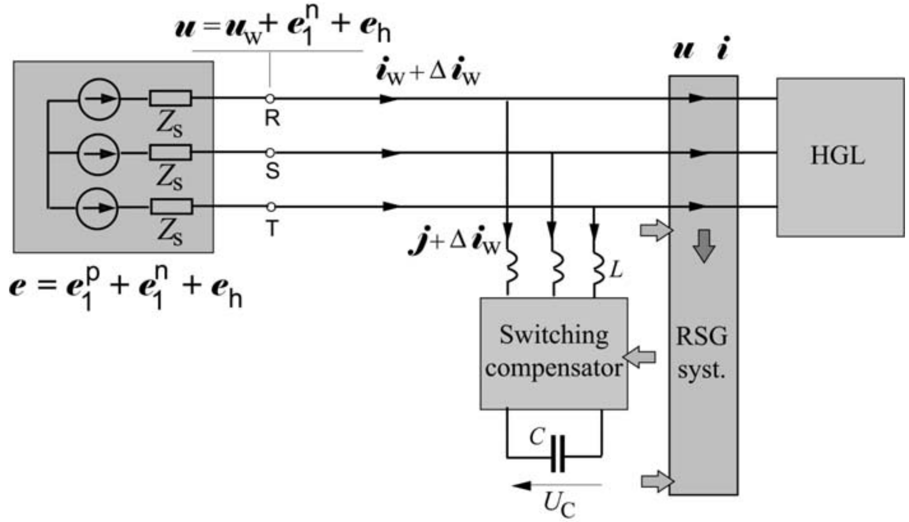
## Switching compensator current

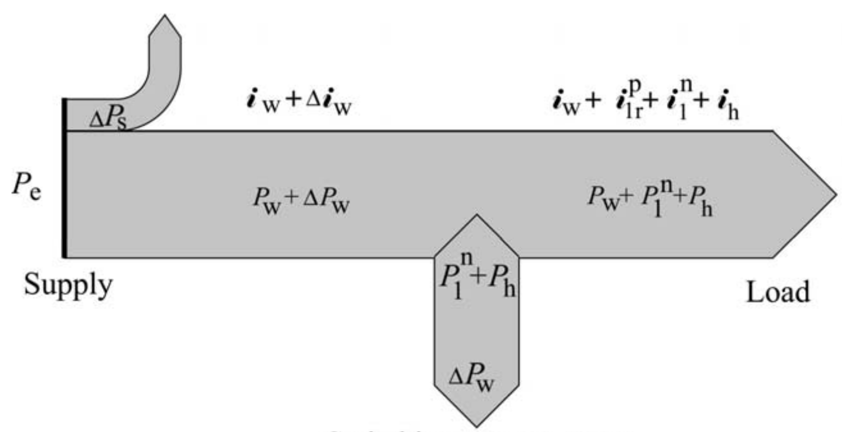
$$\boldsymbol{j} = -\,\boldsymbol{i}_{\mathrm{d}} = -\,(\boldsymbol{i}_{1\mathrm{r}}^{\mathrm{p}} + \boldsymbol{i}_1^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{h}})$$

.

## Compensator current is not orthogonal to the supply voltage

$$(\boldsymbol{u}, \boldsymbol{j}) = (\boldsymbol{u}_{\mathrm{w0}} + \boldsymbol{e}_1^{\mathrm{n}} + \boldsymbol{e}_{\mathrm{h}},\; -\boldsymbol{i}_{1\mathrm{r}}^{\mathrm{p}} - \boldsymbol{i}_1^{\mathrm{n}} - \boldsymbol{i}_{\mathrm{h}}) = -(\boldsymbol{e}_1^{\mathrm{n}}, \boldsymbol{i}_1^{\mathrm{n}}) - (\boldsymbol{e}_{\mathrm{h}}, \boldsymbol{i}_{\mathrm{h}}) = -P_1^{\mathrm{n}} - P_{\mathrm{h}}$$


u = uw+ e1n + eh
u i
iw + Δ iw
R
S
T
Zs
HGL
j+ Δ iw
L
e = e1p + e1n + eh
Switching compensator
RSG syst.
C
UC


ΔPs
iw + Δiw
iw + i1rp + i1n + ih
Pe
Pw + ΔPw
Pw + P1n + Ph
P1n + Ph
Supply
Load
ΔPw
Switching compensator

Should only the working current remain
after compensation
????
is a debatable question

$$||\boldsymbol{i}_{\mathrm{a}}|| < ||\boldsymbol{i}_{\mathrm{w}}||$$

$$||\boldsymbol{i}_{\mathrm{aC}}|| < ||\boldsymbol{i}_{\mathrm{w}}||$$

Energy loss at delivery
is lower when the supply current is reduced to the active current
than
it is reduced to the working current

When the supply voltage
is nonsinusoidal and asymmetrical, however,
then
the active current is also nonsinusoidal and asymmetrical

Question as to compensation to the active or to the working current depends

on which side is the compensator.

It is on the provider or on the consumer side